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RESEARCH MEMORANDUM

AN ANALYTICAL METHOD OF ESTIMATING TURBINE PERFORMANCE

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SUMMARY

A method is developed by which the performance of a turbine
over a range of operating conditions can be analytically estimated
from the blade angles and flow areas. In order to use the method,
certain coefficients that determine the weight flow and friction
losses must be approximated.

The method is used to calculate the performance of the single-
stage turbine of a commercial aircraft gas-turbine engine and the
calculated performance is compared with the performance indicated
by experimental data. For the turbine of the typical example, the
assumed pressure losses and turning angles give a calculated per-
formance that represents the trends of the experimental performance
with reasonable accuracy. The exact agreement between analytical
performance and experimental performance is contingent upon the
proper selection of the blading-loss parameter. A variation of
blading-loss parameter from 0.3 to 0.5 includes most of the
experimental data from the turbine investigated.

INTRODUCTION

The analytical determination of the performance of a gas-
turbine engine under various operating conditions or the prediction
of engine performance at other-than-design conditions requires a
knowledge of the complete performance of each of the engine com-
ponents, especially of the compressor and the turbine (refer-
ences 1 and 2). Knowledge of the component performance charac-
teristics that can be obtained from jet-propulsion-engine investi-
gations is necessarily limited and does not, in general, provide
a solution to the problem of component matching. A performance
test of a compressor or a turbine as a single unit provides all
the necessary information, but such testing of high-speed, high-
capacity units requires costly auxiliary equipment. An analysis
that can be used either with or without engine data to estimate
compressor and turbine performance would be valuable.

An analytical method for estimating turbine performance from the blade angles and flow areas was therefore developed at the NACA Lewis laboratory and is described herein.

The most important prerequisite for application of this analysis is a knowledge of the blade angles and pressure losses of the turbine blades and of the variations of these quantities with changes in angle of incidence and in entrance Mach number. References 3 to 5 present turning angles and losses of typical turbine blades, but their results cannot be generalized to the extent required to obtain the performance of a given blade profile. For the purpose of the turbine analysis, the turning angles and losses and their variations with entrance conditions are therefore assumed, the assumptions being consistent with the information in references 3 to 5. The performance of a turbine of a commercial aircraft gas-turbine engine is determined by means of the analytical method and the results are compared with experimental data.

SYMBOLS

The following symbols and abbreviations are used in this analysis:

A   area perpendicular to velocity, (sq ft)

a,b   blading-loss functions

C   discharge coefficient

D   pitch-line diameter of turbine, (ft)

g   acceleration of gravity, 32.2 (ft/sec^2)

hp   horsepower

i   angle of incidence (angle between direction of approaching fluid and tangent to camberline), (deg)

K   blading-loss parameter

P   total pressure, (lb/sq ft)

p   static pressure, (lb/sq ft)

R   gas constant, 53.3 (ft-lb/($^{\circ}$F)(lb))

r_i inlet velocity ratio, $[U/(V_{3,s} \sin \alpha_{f,s})]$

r_o outlet velocity ratio, $[U/(V_{6,r} \sin \alpha_f)]$

T   total temperature, (°R)

t   static temperature, (°R)

U   rotor-pitch-line velocity, (ft/sec)

V   fluid velocity, (ft/sec)

W   weight flow, (lb/sec)

X   pressure-ratio function, $\left[(p/P)^{\frac{2}{\gamma}} - (p/P)^{\frac{\gamma+1}{\gamma}}\right]^{\frac{1}{2}}$

x,y   jet-deflection parameters

α stator-blade exit angle at pitch line measured from axial
        plane, (deg)

$\alpha_{f,r}$ angle between fluid velocity relative to rotor and axial
        plane at inlet to rotor, (deg)

$\alpha_{f,s}$ angle between fluid absolute velocity and axial plane at
        exit from stator, (deg)

β rotor-blade entrance angle at pitch line measured from axial
        plane, (deg)

γ ratio of specific heats

δ ratio of absolute total pressure to static pressure of
        NACA standard atmosphere at sea level

θ ratio of absolute total temperature to static temperature
        of NACA standard atmosphere at sea level

ϵ angle between fluid absolute velocity and axial plane at
        exit from turbine, (deg)

η_{ad} adiabatic efficiency

ν angle of jet deflection, (deg)

ρ mass density, (slugs/cu ft)

σ rotor-blade exit angle at pitch line measured from axial plane, (deg)

σ_f angle between fluid velocity relative to rotor and axial plane at exit from rotor, (deg)

Subscripts:

a   before cascade

b   at entrance to cascade

c   after cascade

r   with respect to rotor

s   with respect to stator

0   ambient (NACA standard atmosphere at sea level)

1   stator inlet at pitch line

2   stator throat at pitch line

3   stator outlet at pitch line

4   rotor inlet at pitch line

5   rotor throat at pitch line

6   rotor outlet at pitch line

ANALYSIS

Statement of Problem

The quantities commonly employed to define the performance of a turbine and the independent or operating variables are as follows:

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Performance variables	Operating variables
Total-pressure ratio, P_1/P_6	Inlet total pressure, P_1
Total-temperature ratio, T_1/T_6	Inlet total temperature, T_1
Weight flow, W	Rotor-pitch-line velocity, U
Horsepower, hp	Outlet static pressure, p_6
Adiabatic efficiency, η_{ad}	

By means of dimensional analyses or by a method similar to that used in the analysis which follows, it can be shown that the four operating variables can be reduced to two operating parameters. The performance and operating parameters would then be as follows:

Performance parameters	Operating parameters
Total-pressure ratio, $P_{1,s}/P_{6,s}$ (based on axial leaving velocity)	Turbine pressure ratio, $P_{1,s}/p_6$
Total-temperature ratio, $T_{1,s}/T_{6,s}$	Rotor-pitch-line velocity, $U/\sqrt{\theta_{1,s}}$
Weight flow, $W\sqrt{\theta_{1,s}}/\delta_{1,s}$	
Horsepower, $hp/(\sqrt{\theta_{1,s}}\,\delta_{1,s})$	
Adiabatic efficiency, η_{ad}	

The object of this analysis is to determine the value of each of the performance parameters for given values of the two operating parameters.

Outline of Method

For the purpose of the analysis, the turbine pressure ratio $P_{1,s}/p_6$ cannot be used directly because it is a product of the interdependent pressure ratios $P_{1,s}/P_{3,s}$, $P_{3,s}/p_3$, $p_3/P_{4,r}$, $P_{4,r}/P_{6,r}$, and $P_{6,r}/p_6$, which cannot be determined for a given value of $P_{1,s}/p_6$. If instead, values are chosen for $P_{3,s}/p_3$ and $U/\sqrt{\theta_{1,s}}$, and if the pressure losses and turning angles are known or assumed, a step-by-step process can be employed in which

the successive pressures, temperatures, and velocities can be cal-
culated from the principles of fluid mechanics. After the pres-
sures, temperatures, and velocities at the turbine outlet have been
obtained, the performance parameters and also $P_{1,s}/P_6$ can be
computed; the performance parameters can then be plotted as
functions of $P_{1,s}/P_6$ and $U/\sqrt{\theta_{1,s}}$.

Assumptions

Blading losses. - The loss in total pressure that occurs
during passage of fluid through a cascade of blades may, for
convenience of analysis, be separated into two losses: the entry
loss caused by other-than-zero entrance angle and the friction
loss that results from the actual passage through the cascade.

The entry loss can be approximated (fig. 1) by resolving the
entering velocity into components normal and parallel to the blade-
entrance direction and by then assuming that the normal component
is lost; that is, $V_b = V_a \cos i$. This approximation gives entry
losses that are independent of the sign of the angle of incidence.
The losses for positive angles are actually greater than for
corresponding negative angles, but for blading having a solidity
greater than 1.5 the error is probably not too large. For blades
having solidities less than 1.0, however, a better approximation
might be obtained by assuming the normal component to be lost for
positive angles of incidence and no entry losses to occur for
negative angles.

The friction loss can be approximated by

$$\Delta P_{loss} = \frac{2(\gamma-1)}{\gamma} K\left(\frac{1}{2} \rho V^2\right)$$

where K is constant for a particular cascade and $\frac{1}{2} \rho V^2$ is
the average dynamic head of the fluid in the cascade. This
expression for loss (appendix A) can be rewritten as follows:

For the stator

$$P_{3,s} = P_{1,s} \frac{1}{b_3} \tag{1}$$

For the rotor

$$P_{6,r} = P_{4,r} \frac{a_4}{b_6} \qquad\qquad (2)$$

where

$$a = 1 - K\left(\frac{p}{P}\right)^{\frac{1}{\gamma}}\left[1 - \left(\frac{p}{P}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

and

$$b = 1 + K\left(\frac{p}{P}\right)^{\frac{1}{\gamma}}\left[1 - \left(\frac{p}{P}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

A plot of a and b as functions of the pressure ratio p/P for various values of K when γ equals 1.34 is given in figure 2.

Other losses such as blade-tip and separation losses should be small for well-designed turbines operating reasonably close to design conditions and therefore will be neglected in this analysis.

Turning angles. - The angle through which a fluid is turned on passing through a given cascade depends on the angle of incidence, the blade exit angle, and the entrance Mach number. For blading having a solidity of 1.5 or greater, the turning angle so varies with the angle of incidence that the fluid exit angle remains approximately equal to the blade exit angle. The effect of entrance Mach number on the exit angle is small unless sonic velocity exists at the blade throat. In this case, the jet of fluid passes the trailing edge of the blade at a sonic or super-sonic velocity and, if the pressure in the region beyond the cascade is not equal to the pressure at the blade throat, the jet will deflect. The angle of deflection will depend on the pressure ratio in accordance with the Prandtl-Meyer theory. (See, for example, reference 6.) Although a deflection may actually occur at both the upper and lower trailing edges and the flow configuration may be quite complex, it will be assumed for simplicity that the mean deflection is given by the Prandtl-Meyer theory. The angle of jet deflection as a function of the ratio of static-to-total pressure in the region beyond the trailing edge of the blade for sonic throat velocity is calculated from the relations in appendix B and is plotted in figure 3. It will therefore be assumed that the angle at which the fluid leaves the blade is equal

to the difference between the geometrical angle at the pitch line of the trailing edge of the blade and the angle of jet deflection given by the Prandtl-Meyer theory.

Basic Equations

The basic equations to be used can be derived from the definition of total pressure

$$P = p \left[1 + \frac{(\gamma-1) \, V^2}{2\gamma gRt} \right]^{\frac{\gamma}{\gamma-1}} \tag{3}$$

the definition of total temperature

$$T = t \left[1 + \frac{(\gamma-1) \, V^2}{2\gamma gRt} \right] \tag{4}$$

the equation of state

$$p = \rho gRt \tag{5}$$

and the principle of conservation of energy, which can be stated as follows: When heat is neither added nor removed and when the velocity is referred to a set of coordinates that move at the velocity of the blade row, the total temperature remains constant from point to point, whereas the total pressure varies in accordance with viscous losses.

Equations (3) and (4) can be rewritten as

$$p = P \left[1 - \frac{(\gamma-1) \, V^2}{2\gamma gRT} \right]^{\frac{\gamma}{\gamma-1}} \tag{6}$$

and

$$t = T \left[1 - \frac{(\gamma-1) \, V^2}{2\gamma gRT} \right] \tag{7}$$

The weight flow at any point is $W = Cg\rho AV$ and from equations (5) to (7)

$$\rho = \frac{p}{gRt} = \frac{p}{gRT\left[1 - \frac{(\gamma-1)\,V^2}{2\gamma gRT}\right]} = \frac{p}{gRT}\left(\frac{p}{P}\right)^{\frac{\gamma-1}{\gamma}}$$

Also from equation (6)

$$V^2 = \frac{2\gamma gR}{\gamma-1}\,T\left[1 - \left(\frac{p}{P}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{8}$$

so that

$$W = C\,A\,\frac{p}{RT}\left(\frac{p}{P}\right)^{\frac{\gamma-1}{\gamma}}\sqrt{\frac{2\gamma gRT}{\gamma-1}\left[1 - \left(\frac{p}{P}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{1}{2}}}$$

or

$$W = C\,A\,\sqrt{\frac{2\gamma g}{(\gamma-1)R}}\,\frac{P}{\sqrt{T}}\left[\left(\frac{p}{P}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{P}\right)^{\frac{\gamma+1}{\gamma}}\right]^{\frac{1}{2}} \tag{9}$$

Turbine Analysis

The analysis is made for a single-stage partial-reaction turbine having one set of stator blades and one set of rotor blades, each of the blade sets forming a series of convergent nozzles. A schematic diagram of the turbine is shown in figure 4 and the velocity diagram in figure 5. An impulse turbine or a turbine having blade sections that form convergent-divergent nozzles can be considered by making slight alterations to the method.

The quantities obtained from the geometry of the turbine are:

Area of stator throat, A_2

Area of rotor throat, A_5

Pitch-line diameter of turbine, D

Stator-blade exit angle, α

Rotor-blade entrance angle, β

Rotor-blade exit angle, σ

The quantities that must in general be assumed are:

Discharge coefficient, C

Blading-loss parameter, K

Ratio of specific heats, γ

Stator. - When a value for $p_3/P_{3,s}$ is assumed, then b_3 can be obtained from figure 2 and hence $p_3/P_{1,s}$ can be calculated from equation (1),

$$\frac{p_3}{P_{1,s}} = \frac{p_3}{P_{3,s}} \frac{1}{b_3}$$

The weight of gas flowing through the stator from equation (9) is

$$W = C\ A_2 \sqrt{\frac{2\gamma g}{(\gamma-1)R}}\ \frac{P_{2,s}}{\sqrt{T_{2,s}}}\ X_2 \tag{10}$$

where

$$X = \left[\left(\frac{p}{P}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{P}\right)^{\frac{\gamma+1}{\gamma}}\right]^{\frac{1}{2}}$$

In figure 6 X is shown as a function of p/P for two values of γ.

If $\dfrac{P_3}{P_{3,s}} \leq \left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$, the stator is choked and $\dfrac{P_2}{P_{2,s}} = \left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$

Inasmuch as $P_{1,s} = P_{2,s}b_2$ and $T_{1,s} = T_{2,s}$, the weight-flow parameter from equation (10) becomes

$$\frac{W\sqrt{\theta_{1,s}}}{\delta_{1,s}} = \frac{CA_2 P_0}{\sqrt{T_0}\ b_2}\sqrt{\frac{2\gamma g}{(\gamma-1)R}}\left[\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{\frac{1}{2}} \qquad (11)$$

If $\dfrac{P_3}{P_{3,s}} > \left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$, then $P_{3,s} = P_{2,s}$ and $p_3 = p_2$. Then $P_{1,s} = P_{2,s}$, $b_2 = P_{3,s}$ and b_3, $T_{1,s} = T_{2,s} = T_{3,s}$, and the weight-flow parameter is

$$\frac{W\sqrt{\theta_{1,s}}}{\delta_{1,s}} = \frac{CA_2 P_0}{\sqrt{T_0}\ b_3}\sqrt{\frac{2\gamma g}{(\gamma-1)R}}\ X_3 \qquad (12)$$

The velocity leaving the stator, from equation (8), is given by

$$\frac{V_{3,s}}{\sqrt{\theta_{1,3}}} = \sqrt{T_0}\sqrt{\frac{2\gamma g R}{\gamma-1}}\left[1 - \left(\frac{P_3}{P_{3,s}}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{1}{2}} \qquad (13)$$

The deflection of the jet ν_3 as it leaves the stator can be obtained from figure 3 for $p_3/P_{3,s}$. The angle at which the jet leaves the stator is then $\alpha_{f,s} = \alpha - \nu_3$.

Rotor. - A value is assumed for $U/\sqrt{\theta_{1,s}}$ and

$$r_1 = \frac{U/\sqrt{\theta_{1,s}}}{V_{3,s}/\sqrt{\theta_{1,s}}\ \sin\alpha_{f,s}} \qquad (15)$$

From the velocity diagram of figure 5

$$V_{3,r}^2 = \left[(V_{3,s} \sin \alpha_{T,s} - U)^2 + (V_{3,s} \cos \alpha_{T,s})^2\right] = (V_{3,s} \sin \alpha_{T,s})^2 \left[\cot^2 \alpha_{T,s} + (1-r_1)^2\right] \quad (16)$$

and

$$\tan \alpha_{T,r} = \frac{V_3 \sin \alpha_{T,s} - U}{V_3 \cos \alpha_{T,s}} = \tan \alpha_{T,s} (1-r_1) \quad (17)$$

According to the assumption on entry losses,

$$V_{4,r} = V_{3,r} \cos (\alpha_{T,r} - \beta)$$

With equation (16) and the expanded form of $\cos (\alpha_{T,r} - \beta)$, this equation becomes

$$V_{4,r} = V_{3,r} \sin \alpha_{T,s} \left[\cot^2 \alpha_{T,s} + (1-r_1)^2\right]^{\frac{1}{2}} (\cos \alpha_{T,r} \cos \beta + \sin \alpha_{T,r} \sin \beta)$$

or

$$V_{4,r} = V_{3,s} \sin \alpha_{T,s} \left[\cot^2 \alpha_{T,s} + (1-r_1)^2\right]^{\frac{1}{2}} \cos \alpha_{T,r} (\cos \beta + \tan \alpha_{T,r} \sin \beta)$$

Also

$$\cos \alpha_{T,r} = \left(\frac{1}{1 + \tan^2 \alpha_{T,s}}\right)^{\frac{1}{2}}$$

so that finally by use of equation (17)

$$\frac{V_{4,r}}{\sqrt{\theta_{1,s}}} = \frac{V_{3,s}}{\sqrt{\theta_{1,s}}} \cos \alpha_{f,s} \left[\cos \beta + (1-r_1) \tan \alpha_{f,s} \sin \beta \right] \quad (18)$$

In order to find the total temperature with respect to the rotor, equation (4) is used, which gives

$$T_{1,s} = T_{3,s} = t_3 + \frac{(\gamma-1) \, V_{3,s}^2}{2\gamma gR}$$

and

$$T_{3,r} = t_3 + \frac{(\gamma-1) \, V_{3,r}^2}{2\gamma gR}$$

Therefore

$$T_{3,r} = T_{1,s} \left[1 - \frac{(\gamma-1) \left(V_{3,s}^2 - V_{3,r}^2 \right)}{2\gamma gRT_{1,s}} \right]$$

and with equations (16) and (15)

$$\frac{T_{3,r}}{T_{1,s}} = 1 - \frac{(\gamma-1) \, U^2}{2\gamma gRT_{1,s}} \left(\frac{2-r_1}{r_1} \right) \quad (19)$$

Equations (3) and (4) are used to find the total pressure at the entrance to the rotor with respect to the rotor, which give

$$P_{4,r} = p_4 \left[1 + \frac{(\gamma-1) \, V_{4,r}^2}{2\gamma gRt_4} \right]^{\frac{\gamma}{\gamma-1}}$$

and

$$t_4 = T_{4,r} - \frac{(\gamma-1) \, V_{4,r}^2}{2\gamma gR}$$

Then, because $p_4 = p_3$ and $T_{4,r} = T_{3,r}$

$$\frac{p_3}{P_{4,r}} = \left[1 - \left(\frac{V_{4,r}}{\sqrt{T_{1,s}}}\right)^2 \left(\frac{\sqrt{T_{1,s}}}{\sqrt{T_{3,r}}}\right)^2 \frac{(\gamma-1)}{2\gamma gR}\right]^{\frac{\gamma}{\gamma-1}} \tag{20}$$

The weight-flow parameter at the rotor throat can be written as

$$\frac{W\sqrt{\theta_{5,r}}}{\delta_{5,r}} = \frac{W\sqrt{\theta_{1,s}}}{\delta_{1,s}} \left(\frac{T_{3,r}}{T_{1,s}}\right)^{\frac{1}{2}} \left(\frac{T_{5,r}}{T_{3,r}}\right)^{\frac{1}{2}} \frac{P_{1,s}}{P_3} \frac{P_3}{P_{4,r}} \frac{P_{4,r}}{P_{5,r}}$$

where from equation (9)

$$\frac{W\sqrt{\theta_{5,r}}}{\delta_{5,r}} = \frac{P_0}{\sqrt{T_0}} C A_5 \sqrt{\frac{2\gamma g}{(\gamma-1) R}} X_5$$

Therefore, because $T_{3,r} = T_{5,r}$,

$$X_5 = \frac{\dfrac{W\sqrt{\theta_{1,s}}}{\delta_{1,s}} \left(\dfrac{T_{3,r}}{T_{1,s}}\right)^{\frac{1}{2}} \dfrac{P_{1,s}}{P_3} \dfrac{P_3}{P_{4,r}} \dfrac{P_{4,r}}{P_{5,r}}}{\dfrac{P_0}{\sqrt{T_0}} C A_5 \sqrt{\dfrac{2\gamma g}{(\gamma-1) R}}} \tag{21}$$

The only unknown quantity on the right-hand side of equation (21) is the pressure-loss term $P_{4,r}/P_{5,r}$ but in order to determine this term from the loss equation

$$\frac{P_{4,r}}{P_{5,r}} = \frac{b_5}{a_4}$$

b_5 must be known. The concept b_5 is a function of $p_5/P_{5,r}$ and hence of X_5. A trial-and-error process must therefore be used.

A value of $p_5/P_{5,r}$ such that

$$\frac{P_3}{P_{4,r}} > \frac{p_5}{P_{5,r}} \geq \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

can be chosen as a first approximation. Then b_5, $P_{4,r}/P_{5,r}$, X_5, and finally $p_5/P_{5,r}$ can be obtained. From this second approximation, a second value of b_5 can be obtained and the process repeated. Convergence is rapid and the second approximation will usually be sufficiently accurate.

The maximum value that X_5 can have is

$$\left[\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{\frac{1}{2}}$$

and if the value of X_5 from equation (21) is greater than the maximum, one of the original assumptions (that is, $p_3/P_{3,s}$ or $U/\sqrt{\theta_{1,s}}$) must have been inadmissable.

If the rotor is not choked, the static pressure in the annulus following the rotor p_6 is equal to the static pressure at the rotor throat p_5. When the rotor is choked, p_6 can be less than p_5 and consequently $p_6/P_{6,r}$ can be less than $p_5/P_{5,r}$. In this case a value can be assumed for $p_6/P_{6,r}$ and b_6 can be obtained from figure 2. This procedure can be summarized as follows:

For

$$X_5 < \left[\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{\frac{1}{2}}$$

$$\frac{p_6}{P_{6,r}} = \frac{p_5}{P_{5,r}}$$

and

$$b_6 = b_5$$

For

$$x_5 = \left[\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{\frac{1}{2}}$$

$$\frac{p_6}{P_{6,r}} \leq \frac{p_5}{P_{5,r}}$$

and

$$b_6 \geq b_5$$

The velocity leaving the rotor with respect to the rotor is given by

$$\frac{V_{6,r}}{\sqrt{\theta_{3,r}}} = \sqrt{\frac{2\gamma gRT_0}{\gamma-1}} \left[1 - \left(\frac{p_6}{P_{6,r}}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{1}{2}} \tag{22}$$

inasmuch as $T_{6,r} = T_{3,r}$

The jet deflection at the rotor outlet ν_6 is obtained from figure 3 for $p_6/P_{6,s}$. The angle at which the fluid leaves the rotor is then

$$\sigma_f = \sigma - \nu_6 \tag{23}$$

From figure 5 the absolute velocity downstream of the rotor is

$$V_{6,s} = \sqrt{(V_{6,r} \sin \sigma_f - U)^2 + (V_{6,r} \cos \sigma_f)^2}$$

or

$$V_{6,s} = V_{6,r} \sin \sigma_f \left[\cot^2 \sigma_f + (1-r_0)^2\right]^{\frac{1}{2}} \tag{24}$$

where

$$r_o = \frac{U/\sqrt{\theta_{1,s}}}{V_{6,r}/\sqrt{\theta_{1,s}}\,\sin\sigma_f} \tag{25}$$

The absolute exit angle θ can be obtained from

$$\tan\theta = \frac{V_{6,r}\,\sin\sigma_f - U}{V_{6,r}\,\cos\sigma_f}$$

or

$$\tan\theta = \tan\sigma_f\,(1-s) \tag{26}$$

The total temperature downstream of the rotor can be obtained by using equation (4) to give

$$T_{6,s} = t_6 + \frac{(\gamma-1)\,V_{6,s}^{2}}{2\gamma gR}$$

and

$$T_{3,r} = T_{6,r} = t_6 + \frac{(\gamma-1)\,V_{6,r}^{2}}{2\gamma gR}$$

or

$$T_{6,s} = T_{3,r} - \frac{(\gamma-1)(V_{6,r}^{2} - V_{6,s}^{2})}{2\gamma gR}$$

Then from equations (24) and (25)

$$\frac{T_{6,s}}{T_{3,r}} = 1 - \frac{(\gamma-1)\,U^2}{2\gamma gRT_{3,r}}\left(\frac{2-r_o}{r_o}\right) \tag{27}$$

Because, in general, the tangential component of the velocity leaving the rotor cannot be converted into useful thrust, the total

pressure downstream should be calculated from the axial component
of the exit velocity; that is

$$\frac{P_{6,s}}{p_6} = \left[1 + \frac{(\gamma-1)(V_{6,r}\cos\sigma_f)^2}{2\gamma gRt_6}\right]^{\frac{\gamma}{\gamma-1}}$$

or, because

$$t_6 = T_{3,r} - \frac{(\gamma-1)V_{6,r}^2}{2\gamma gR}$$

$$\frac{P_{6,s}}{p_6} = \left[\frac{1 - \left(\frac{V_{6,r}}{\sqrt{T_{3,r}}}\sin\sigma_f\right)^2\frac{\gamma-1}{2\gamma gR}}{1 - \left(\frac{V_{6,r}}{\sqrt{T_{3,r}}}\right)^2\frac{\gamma-1}{2\gamma gR}}\right]^{\frac{\gamma}{\gamma-1}} \qquad (28)$$

Performance parameters. - The fluid velocity, the pressure, and
the temperature at any point in the turbine are now known. The
turbine performance can be evaluated by combining these quantities
to form the following parameters:

$$\frac{T_{6,s}}{T_{1,s}}, \quad \frac{p_6}{P_{1,s}}, \quad \frac{P_{6,s}}{P_{1,s}}, \quad \eta_{ad}, \quad \frac{hp}{\sqrt{\theta_{1,s}}\,\delta_{1,s}}$$

The ratio of total temperatures can be rewritten as

$$\frac{T_{6,s}}{T_{1,s}} = \left(\frac{T_{6,s}}{T_{3,r}}\right)\left(\frac{T_{3,r}}{T_{1,s}}\right)$$

or, with equations (27) and (19)

$$\frac{T_{6,s}}{T_{1,s}} = \left[1 - \frac{(\gamma-1)U^2}{2\gamma gRT_{1,s}}\left(\frac{2-r_o}{r_o}\right)\left(\frac{T_{1,s}}{T_{3,r}}\right)\right]\left[1 - \frac{(\gamma-1)U^2}{2\gamma gRT_{1,s}}\left(\frac{2-r_i}{r_i}\right)\right]^2$$

Substitution for $T_{1,s}/T_{3,r}$ from equation (19) finally gives

$$\frac{T_{6,s}}{T_{1,s}} = 1 - \frac{(\gamma-1)U^2}{2\gamma g R T_{1,s}}\left(\frac{2-r_i}{r_i} + \frac{2-r_o}{r_o}\right) \tag{29}$$

The other performance parameters can be obtained as follows:

$$\frac{P_{6,s}}{P_{1,s}} = \left(\frac{p_3}{P_{1,s}}\right)\left(\frac{P_{4,r}}{p_3}\right)\left(\frac{P_{6,r}}{P_{4,r}}\right)\left(\frac{p_6}{P_{6,r}}\right) \tag{30}$$

$$\frac{P_{6,s}}{P_{1,s}} = \left(\frac{p_6}{P_{1,s}}\right)\left(\frac{P_{6,s}}{p_6}\right) \tag{31}$$

$$\eta_{ad} = \frac{1 - \dfrac{T_{6,s}}{T_{1,s}}}{1 - \left(\dfrac{P_{6,s}}{P_{1,s}}\right)^{\frac{\gamma-1}{\gamma}}} \tag{32}$$

$$hp = \frac{\gamma}{\gamma-1} R T_{1,s}\left(1 - \frac{T_{6,s}}{T_{1,s}}\right)W\left(\frac{1}{550}\right)$$

$$\frac{hp}{\sqrt{\theta_{1,s}}\,\delta_{1,s}} = \frac{T_0 R}{(\gamma-1)\,550}\left(1 - \frac{T_{6,s}}{T_{1,s}}\right)\frac{W\sqrt{\theta_{1,s}}}{\delta_{1,s}} \tag{33}$$

TYPICAL EXAMPLE OF METHOD

As an example of the use of the method and of the results that can be obtained, the performance for the turbine of a commercial aircraft gas-turbine engine will be calculated and the calculated

performance will be compared with the experimental. The geometrical properties of the turbine are as follows:

$$A_2 = 0.852 \text{ square feet} \qquad \alpha = 62^\circ$$

$$A_5 = 1.26 \text{ square feet} \qquad \beta = 24^\circ$$

$$D = 1.833 \text{ feet} \qquad \sigma = 47^\circ$$

The values of C, K, and γ for this example are assumed as follows:

$$C = 0.98$$

$$K = 0.30$$

$$\gamma_s = 1.33$$

$$\gamma_r = 1.34$$

With these values the method is as follows:

(a) Assume $p_3/P_{3,s}$ and $U/\sqrt{\theta_{1,s}}$.

(b) Find b_3 from figure 2 and ν_3 from figure 3.

(c) $p_3/P_{1,s} = p_3/P_{3,s} \left(\dfrac{1}{b_3}\right)$

(d) If $p_3/P_3 \geq 0.539$, $p_2/P_{2,s} = p_3/P_{3,s}$;

     if $p_3/P_{3,s} < 0.539$, $p_2/P_{2,s} = 0.539$.

(e) Find b_2 from figure 2.

(f) $X_2 = \left[(p_2/P_{2,s})^{1.504} - (p_2/P_{2,s})^{1.752}\right]^{\frac{1}{2}}$

(g) $\dfrac{W\sqrt{\theta_{1,s}}}{\delta_{1,s}} = \dfrac{1.842\ P_0}{\sqrt{T_0}\ b_2}\ X_2$

(h) $V_{3,s}/\sqrt{\theta_{1,s}} = 117.6\sqrt{T_0}\left[1 - \left(\dfrac{p_3}{P_{3,s}}\right)^{0.248}\right]^{\frac{1}{2}}$

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(i) $\alpha_{f,s} = 62 - \nu_3$

(j) $r_i = \dfrac{U/\sqrt{\theta_{1,s}}}{V_{3,s}/\sqrt{\theta_{1,s}} \ \sin \alpha_{f,s}}$

(k) $T_{3,r}/T_{1,s} = 1 - \left(\dfrac{1}{116.3}\dfrac{U}{\sqrt{T_{1,s}}}\right)^2 \left(\dfrac{2-r_i}{r_i}\right)$

(l) $\dfrac{V_{4,r}}{\sqrt{\theta_{1,s}}} = \dfrac{V_{3,s}}{\sqrt{\theta_{1,s}}} \cos \alpha_{f,s} \left[\cos 24° + (1-r)\ \tan \alpha_{f,s} \ \sin \Delta m \ 24°\right]$

(m) $p_3/P_{4,r} = \left[1 - \left(\dfrac{1}{116.3}\dfrac{V_{4,r}}{\sqrt{T_{1,s}}}\right)^2 \dfrac{T_{1,s}}{T_{3,r}}\right]^{3.941}$

(n) Find a_4 from figure 2 and assume b_5.

(o) $X_5 = \dfrac{1}{2.695}\dfrac{W\sqrt{\theta_{1,s}}}{\delta_{1,s}}\left(\dfrac{T_{3,r}}{T_{1,s}}\right)^{\frac{1}{2}}\left(\dfrac{P_{1,s}}{p_3}\right)\left(\dfrac{p_3}{P_{4,r}}\right)\left(\dfrac{b_5}{a_4}\right)\left(\dfrac{\sqrt{T_0}}{P_0}\right)$

(p) Find $p_5/P_{5,r}$ from figure 6.

(q) Find b_5 from figure 2 and check with assumed b_5.

(r) If $p_5/P_{5,r} > 0.537$, $\dfrac{p_6}{P_{6,r}} = \dfrac{p_5}{P_{5,r}}$; if $p_5/P_{5,r} = 0.537$,

assume $p_6/P_{6,r} \leq 0.537$.

(s) $V_{6,r}/\sqrt{T_{3,r}} = 116.3 \left[1 - \left(\dfrac{p_6}{P_{6,r}}\right)^{0.254}\right]^{\frac{1}{2}}$

(t) Find ν_6 from figure 3 and b_6 from figure 2.

(u) $\sigma_f = 47 - \nu_6$

(v) $r_0 = \dfrac{U/\sqrt{\theta_{1,s}}}{\dfrac{V_{6,r}}{\sqrt{T_{3,r}}}\dfrac{\sqrt{T_{3,r}}}{\sqrt{\theta_{1,r}}}\sin\sigma_f}$

(w) $T_{6,s}/T_{3,r} = 1 - \left(\dfrac{1}{116.3}\dfrac{U}{\sqrt{T_{1,s}}}\right)^2\left(\dfrac{T_{1,s}}{T_{3,s}}\right)\left(\dfrac{2-r_0}{r_0}\right)$

(x) $P_{6,s}/P_6 = \left[\dfrac{1 - \left(\dfrac{1}{116.3}\dfrac{V_{6,r}}{\sqrt{T_{3,r}}}\sin\sigma_f\right)^2}{1 - \left(\dfrac{1}{116.3}\dfrac{V_{6,r}}{\sqrt{T_{3,r}}}\right)^2}\right]^{3.941}$

The performance parameters are:

$$T_{6,s}/T_{1,s} = 1 - \left(\frac{1}{116.3}\frac{U}{\sqrt{T_{1,s}}}\right)^2\left(\frac{2-r_1}{r_1} + \frac{2-r_0}{r_0}\right)$$

$$P_6/P_{1,s} = (P_3/P_{1,s})\,(P_{4,r}/P_3)\,(a_4/b_6)\,(p_6/P_{6,r})$$

$$P_{6,s}/P_{1,s} = (p_6/P_{1,s})\,(P_{6,s}/p_6)$$

$$\eta_{ad} = \frac{1 - (T_{6,s}/T_{1,s})}{1 - \left(\dfrac{P_{6,s}}{P_{1,s}}\right)^{0.251}}$$

$$\frac{hp}{\sqrt{\theta_{1,s}}\,\delta_{1,s}} = 0.3628\,T_0\left(1 - \frac{T_{6,s}}{T_{1,s}}\right)\frac{W\sqrt{\theta_{1,s}}}{\delta_{1,s}}$$

A summary of the calculations is given in table I.

For step (r), it is necessary to find the value of $U/\sqrt{\theta_{1,s}}$ for which $p_5/P_{5,r} = 0.537$, that is, for which the rotor is choked and $X_5 = 0.241$. This value can be found most simply by plotting X_5 against $U/\sqrt{\theta_{1,s}}$ (fig. 7) and finding the value of $U/\sqrt{\theta_{1,s}}$ that corresponds to $X_5 = 0.241$.

Because the final performance parameters are to be obtained as functions of $P_{1,s}/p_6$ for constant $U/\sqrt{\theta_{1,s}}$, it is desirable to have the choking values of $U/\sqrt{\theta_{1,s}}$ correspond to the initially chosen values of $U/\sqrt{\theta_{1,s}}$. This correspondence can be accomplished by plotting the choking values of $U/\sqrt{\theta_{1,s}}$ against $p_3/P_{3,s}$ (fig. 8) and finding the values of $p_3/P_{3,s}$ that correspond to the desired values of $U/\sqrt{\theta_{1,s}}$.

From figure 8, the rotor-choking values of $p_3/P_{3,s}$ that correspond to values of $U/\sqrt{\theta_{1,s}}$ of 455 and 638 are 0.5245 and 0.5650, respectively. The performance calculations for these values of $p_3/P_{3,s}$ are summarized in table II.

The performance curves of figure 9 are plotted from the values in tables I and II.

DISCUSSION OF PERFORMANCE CURVES

Trends of Calculated Performance

Choking of the turbine, in either the stator or the rotor, is one of the most important factors affecting turbine performance. The conditions for stator and rotor choking are obtained from table II for the turbine of the typical example and are given in figure 10. The turbine pressure ratio at which the stator chokes increases with increasing rotor-speed parameter whereas the pressure ratio for rotor choking is almost independent of the speed parameter. For values of the rotor-speed parameter greater than 525 the stator cannot choke.

Another important consideration is the variation of inlet and outlet velocity ratios. These ratios determine the rotor entry- and exit-whirl losses and affect mainly the adiabatic efficiency.

When the fluid enters the rotor at the blade entrance angle, the
inlet velocity ratio from equation (17) is

$$r_i = 1 - \frac{\tan 24^{\circ}}{\tan 62^{\circ}} = 0.763$$

For axial outlet velocity (that is, no whirl), the outlet velocity
ratio r_0 is equal to 1.00. As is shown in figure 11, the speed
parameter and pressure ratio for which both of these conditions are
satisfied are 636 and 2.55, respectively.

The variation of the temperature-ratio parameter with turbine
pressure ratio is shown in figure 9(a). The temperature-ratio
parameter is directly proportional to the change of the whirl
component of the fluid as it passes through the rotor. For turbine
pressure ratios less than the pressure ratio for rotor choking
(fig. 10), both the entering and leaving whirl components increase
with pressure ratio. For turbine pressure ratios greater than the
rotor-choking pressure ratio, however, the entering whirl component
is constant and, because of the jet deflection, the leaving whirl
component is maintained almost constant even though the exit
velocity increases with increasing pressure ratio. Hence for any
given rotor-speed parameter, the temperature-ratio parameter
increases with increasing turbine pressure ratio until the rotor
chokes. At pressure ratios higher than choking, the temperature-
ratio parameter remains relatively constant.

The total-pressure ratio (fig. 9(b)) is a function mainly of
the turbine pressure ratio, the rotor-speed parameter having a very
small effect. It should be remembered that for this analysis the
downstream total pressure $P_{6,s}$ has been obtained from the axial
component on the downstream velocity rather than the total down-
stream velocity.

The adiabatic efficiency given in figure 9(c) is also dependent
on the definition of downstream total pressure. The greatest
efficiency is obtained for a speed parameter of 636 and a pressure
ratio of approximately 2.55, which is in accordance with the
previous discussion of the effect of the velocity ratios on adia-
batic efficiency.

The weight-flow parameter (fig. 9(d)) increases with turbine
pressure ratio until the turbine chokes, and then remains constant.
For the two highest speed parameters (636 and 546), the weight flow

is limited by rotor choking, whereas for lower speed parameters the weight flow is limited by stator choking. The greatest weight flow is slightly less for rotor-choking speed parameters than for stator-choking speed parameters.

The variation of the horsepower parameter is shown in figure 9(e). The horsepower parameter is proportional to the product of the temperature-ratio parameter and the weight-flow parameter, both of which increase with turbine pressure ratio for pressure ratios less than 2. For pressure ratios between 2 and 3, the temperature-ratio parameter continues to increase but the weight-flow parameter is almost constant so that the horsepower parameter continues to increase, though not so rapidly as for pressure ratios less than 2. For pressure ratios greater than 3, the weight-flow parameter is constant, the temperature-ratio parameter is almost constant, and the horsepower parameter is almost constant.

The effect of the choice of the blading-loss parameter K on the performance parameters is shown in figure 12. The effect can be seen to consist mainly of a vertical shifting of the curves, each curve retaining its general shape independently of the blading-loss parameter. For a given turbine pressure ratio, increasing the blading-loss parameter by 0.1 results in an increase in the total-pressure ratio of approximately 0.5 percent and a decrease in the temperature-ratio parameter and the adiabatic efficiency of approximately 2.0 percent.

Comparison with Experimental Results

The turbine of the typical example has been operated at the NACA Lewis laboratory as a part of a gas-turbine engine and the resulting data have been used to obtain the turbine performance parameters. The variation of temperature-ratio parameter and adiabatic efficiency with rotor-speed parameter is presented in figure 13 for values of the turbine total-pressure ratio ranging from 1.71 to 2.34. Curves of calculated performance are also presented for values of the blading-loss parameter K of 0.3 and 0.5. In general, the temperature-ratio-parameter comparison indicates a blading-loss parameter of about 0.3 except at a pressure ratio of 2.34 where the blading-loss parameter shows some increase. The trend of the efficiency comparison is toward a blading-loss parameter somewhat higher than 0.3, but in general less than 0.5. The trend of a higher indicated blading-loss parameter on the basis of the efficiency comparison is probably due to the additional error involved in the use of an

experimentally determined pressure ratio to calculate the measured
efficiency. The importance of additional investigations of condi-
tions that influence the blading-loss parameter is clearly evident
inasmuch as the degree of agreement between theory and experiment
is contingent on the proper choice of this parameter. For the
turbine investigated, a blading-loss parameter of 0.5 would be
slightly higher than the experiments indicate and the complete
engine would actually perform somewhat better than would be
predicted by a theoretical matching study.

CONCLUSIONS

From a theoretical investigation of the performance of the
turbine component of a commercial aircraft gas-turbine engine and
a comparison of the performance calculated by means of the
analytical method and the experimental performance, it is concluded
that:

1. For the turbine of the typical example, the assumed pres-
sure losses and turning angles give a calculated performance that
represents the trends of the experimental performance with
reasonable accuracy.

2. The agreement between analytical performance and experi-
mental performance is contingent upon the proper selection of
blading-loss parameter. The experimental data indicate that for
the turbine of the typical example the blading-loss parameter
varied from 0.3 to 0.5 for most of the range investigated.

3. The methods of analysis should be applicable to any turbine
with reasonably well designed blading, although the same coeffi-
cient will not necessarily apply. For turbines with poor blading
design, the pressure losses and the turning angles will be
difficult to evaluate and the trends, as well as the magnitudes of
the calculated performance, may be incorrect.

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        Cleveland, Ohio.

APPENDIX A

BLADE FRICTION LOSSES

The loss in total pressure due to friction effects that occur when a fluid passes through a cascade depends on the design of the particular cascade, on the Reynolds number, and on the average dynamic head in the cascade. For a particular cascade, however, the effect of a change in the Reynolds number caused by a change in operating conditions should be small even for a reasonably wide range of operating conditions. The loss can therefore be approximated by

$$\Delta P_{loss} = \frac{2(\gamma-1)}{\gamma} K \left(\frac{1}{2} \rho V^2 \right) \tag{34}$$

where K is a constant for the particular cascade and $\frac{1}{2} \rho V^2$ is the average dynamic head in the cascade.

From equations (5), (6), and (7)

$$\frac{1}{2} \rho V^2 = \frac{\gamma}{\gamma-1} P \left[1 - \left(\frac{p}{P} \right)^{\frac{\gamma-1}{\gamma}} \right] \left(\frac{p}{P} \right)^{\frac{1}{\gamma}}$$

The average dynamic head in the cascade of figure 1 is then approximately

$$\frac{\gamma}{2(\gamma-1)} \left\{ P_b \left(\frac{p_b}{P_b} \right)^{\frac{1}{\gamma}} \left[1 - \left(\frac{p_b}{P_b} \right)^{\frac{\gamma-1}{\gamma}} \right] + P_c \left(\frac{p_c}{P_c} \right)^{\frac{1}{\gamma}} \left[1 - \left(\frac{p_c}{P_c} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\} \tag{35}$$

The pressure loss then becomes

$$\Delta P_{loss} = P_b - P_c = K \left\{ P_b \left(\frac{p_b}{P_b} \right)^{\frac{1}{\gamma}} \left[1 - \left(\frac{p_b}{P_b} \right)^{\frac{\gamma-1}{\gamma}} \right] + P_c \left(\frac{p_c}{P_c} \right)^{\frac{1}{\gamma}} \left[1 - \left(\frac{p_c}{P_c} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}$$

or

$$P_b \quad a = P_c \quad b \tag{36}$$

where

$$a = 1 - K \left(\frac{p_b}{P_b}\right)^{\frac{1}{\gamma}} \left[1 - \left(\frac{p_b}{P_b}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{37}$$

$$b = 1 + K \left(\frac{p_c}{P_c}\right)^{\frac{1}{\gamma}} \left[1 - \left(\frac{p_c}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

The variation of the blading-loss parameter with the ratio of static-to-total pressure for several values of K is shown in figure 2.

For the stator the entrance velocity is small; then

$$a \approx 1$$

and equation (36) becomes

$$P_{1,s} = P_{3,s} \, b_3$$

For the rotor, equation (36) becomes

$$P_{4,r} \, a_4 = P_{6,r} \, b_6$$

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APPENDIX B

JET DEFLECTION

The expansion of a uniform, two-dimensional, frictionless stream of gas flowing around a corner at sonic or supersonic speeds has been investigated by Prandtl and Meyer. The analysis of reference 6 shows that the jet deflection is given by

$$\nu = x - y \tag{38}$$

$$\left(\frac{p}{P}\right)^{\frac{\gamma-1}{\gamma}} = \frac{1}{\gamma+1}\left(1 + \cos 2\sqrt{\frac{\gamma-1}{\gamma+1}}\, x\right) \tag{39}$$

$$\tan y = \sqrt{\frac{\gamma+1}{\gamma-1}}\, \tan\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\, x\right) \tag{40}$$

where ν is the angle of jet deflection and p/P is the ratio of static-to-total pressure in the region beyond the trailing edge. For the purpose of this discussion, x and y can be regarded as parameters. A value is assumed for x, the concepts p/P and y are computed from equations (39) and (40), respectively, and ν is obtained from equation (38). Then ν is known as a function of p/P. (See fig. 3.)

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TABLE I - SUMMARY OF CALCULATIONS FOR UNCHOKED TURBINE ROTOR OF TYPICAL EXAMPLE

$\boxed{K,\ 0.3;\ 0,\ 0.98}$

$\dfrac{p_3}{p_{3,s}}$	$\dfrac{p_3}{p_{1,s}}$	$\dfrac{W\sqrt{\theta_{1,s}}}{\delta_{1,s}}$	$\dfrac{V_{3,s}}{\sqrt{T_{1,s}}}$	$\alpha_{f,s}$	$\dfrac{U}{\sqrt{\theta_{1,s}}}$	r_1	$\dfrac{T_{3,r}}{T_{1,s}}$	$\dfrac{p_3}{p_{4,r}}$	$\dfrac{p_5}{p_{5,r}}$	$\dfrac{p_6}{p_{6,r}}$	$\dfrac{p_{6,r}}{p_{4,r}}$	$\dfrac{V_{6,s}}{\sqrt{T_{3,r}}}$	r_0	$\dfrac{T_{6,s}}{T_{1,s}}$	$\dfrac{p_{6,s}}{p_6}$	$\dfrac{p_{1,s}}{p_6}$	$\dfrac{p_{1,s}}{p_{6,s}}$	η_{ad}	$\dfrac{hp}{\delta_{1,s}\sqrt{\theta_{1,s}}}$
0.600	0.5854	39.36	40.58	62.0	273.2	0.3349	0.9471	0.7967	0.740	0.740	0.9680	31.55	0.5345	0.9179	1.153	1.900	1.646	0.689	607.8
					455.4	.5582	.9236	.8358	.709	.709	.9695	33.66	.8456	.8833	1.178	2.078	1.764	.870	864.2
					637.6	.7815	.9096	.8737	.664	.664	.9695	36.55	1.0980	.8620	1.217	2.319	1.906	.913	1021.8
0.560	0.5455	39.52	43.06	62.0	455.4	0.5260	0.9171	0.8089	0.654	0.654	0.9645	37.18	0.7679	0.8697	1.225	3.350	1.919	0.853	968.8
					614.8	.7107	.9021	.8437	.537	.537	.9620	44.45	.8743	.8326	1.382	2.993	2.215	.914	1243.6
0.540	0.5258	39.54	44.27	62.0	273.2	0.3070	0.9413	0.7558	0.682	0.682	0.9620	35.39	0.4778	0.9074	1.200	2.182	1.819	0.652	688.8
					455.4	.5116	.9140	.7948	.608	.608	.9620	40.06	.7141	.8607	1.270	2.581	2.065	.844	1036.3
					525.7	.5884	.9062	.8103	.537	.537	.9590	44.45	.7431	.8401	1.352	2.990	2.212	.875	1190.1
0.523	0.5088	39.54	45.34	61.8	273.2	0.3003	0.9398	0.7395	0.660	0.660	0.9610	36.81	0.4698	0.9041	1.219	2.290	1.877	0.648	713.4
					448.6	.4930	.9123	.7795	.537	.537	.9585	44.45	.6344	.8505	1.352	2.979	2.201	.822	1112.0
0.500	0.4859	39.54	45.74	61.3	273.2	0.2927	0.9379	0.7184	0.621	0.621	0.9590	39.29	0.4312	0.8992	1.241	2.484	2.001	0.623	750.0
					364.3	.3902	.9219	.7392	.537	.537	.9562	44.45	.5125	.8670	1.352	2.962	2.191	.736	988.6

TABLE II - SUMMARY OF CALCULATIONS FOR CHOKED TURBINE ROTOR OF TYPICAL EXAMPLE

$\boxed{K,\ 0.3;\ C,\ 0.98}$

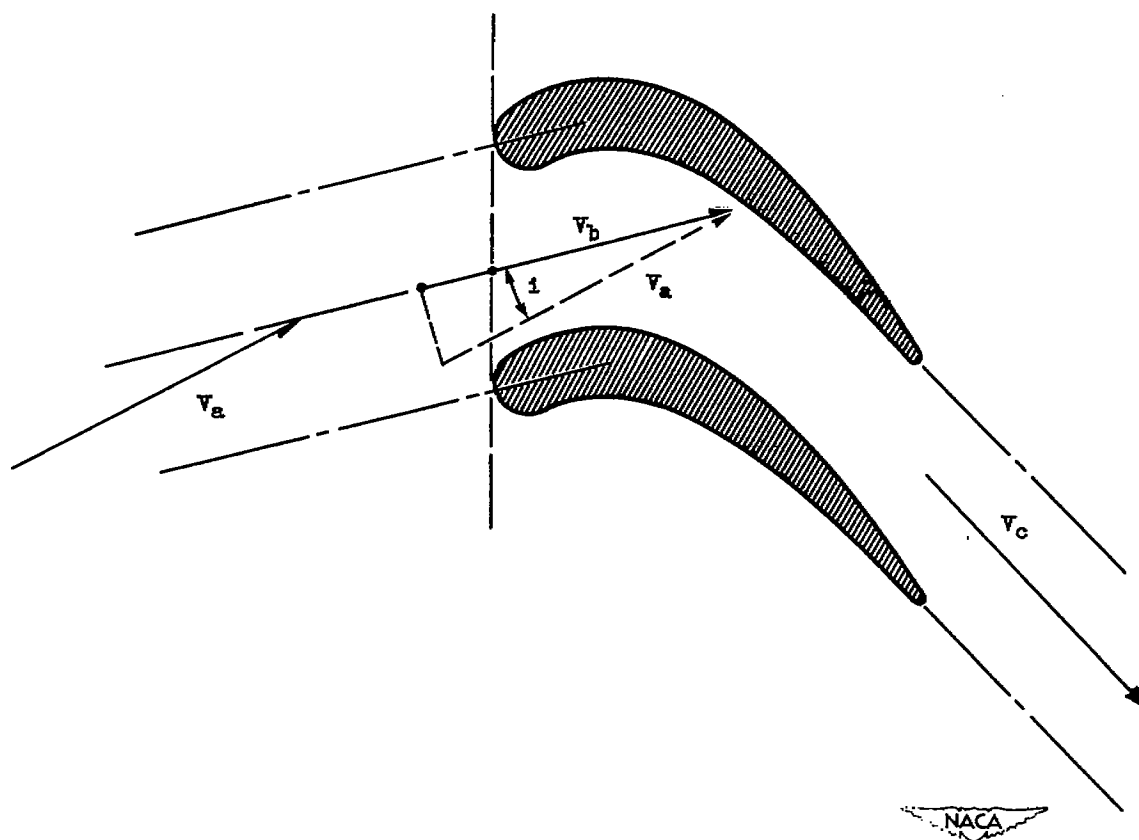| $\dfrac{p_3}{p_{3,s}}$	$\dfrac{p_3}{p_{1,s}}$	$\dfrac{W\sqrt{\theta_{1,s}}}{\delta_{1,s}}$	$\dfrac{V_{3,s}}{\sqrt{T_{1,s}}}$	$\alpha_{f,s}$	$\dfrac{U}{\sqrt{\theta_{1,s}}}$	r_1	$\dfrac{T_{3,r}}{T_{1,s}}$	$\dfrac{p_3}{p_{4,r}}$	$\dfrac{p_5}{p_{5,r}}$	$\dfrac{p_6}{p_{6,r}}$	$\dfrac{p_{6,r}}{p_{4,r}}$	$\dfrac{V_{6,s}}{\sqrt{T_{3,r}}}$	r_0	$\dfrac{T_{6,s}}{T_{1,s}}$	$\dfrac{p_{6,s}}{p_6}$	$\dfrac{p_{1,s}}{p_6}$	$\dfrac{p_{1,s}}{p_{6,s}}$	η_{ad}	$\dfrac{hp}{\delta_{1,s}\sqrt{\theta_{1,s}}}$
0.5650	0.5506	39.49	42.74	62.0	637.6	0.7420	0.9017	0.8523	0.537	0.537	0.9625	44.46	0.9069	0.8319	1.3514	2.996	2.214	0.9185	1314.4
										.500	.9610	46.73	.8727	.8269	1.4151	3.220	2.280	.9169	1353.5
										.450	.9600	49.83	.8441	.8224	1.5311	3.581	2.340	.9143	1388.6
										.400	.9590	53.00	.8282	.8198	1.6966	4.038	2.379	.9125	1409.4
0.5245	0.5103	39.54	45.23	61.8	455.4	0.5017	0.9117	0.7822	0.537	0.537	0.9625	44.46	0.6442	0.8495	1.3514	2.980	2.203	0.8278	1177.0
										.500	.9610	46.73	.6200	.8459	1.4151	3.200	2.266	.8214	1205.0
										.450	.9600	49.83	.5997	.8427	1.5311	3.565	2.328	.8150	1230.5
										.400	.9590	53.00	.5883	.8408	1.6966	4.012	2.366	.8109	1245.0

Figure 1. - Blading-loss diagram.
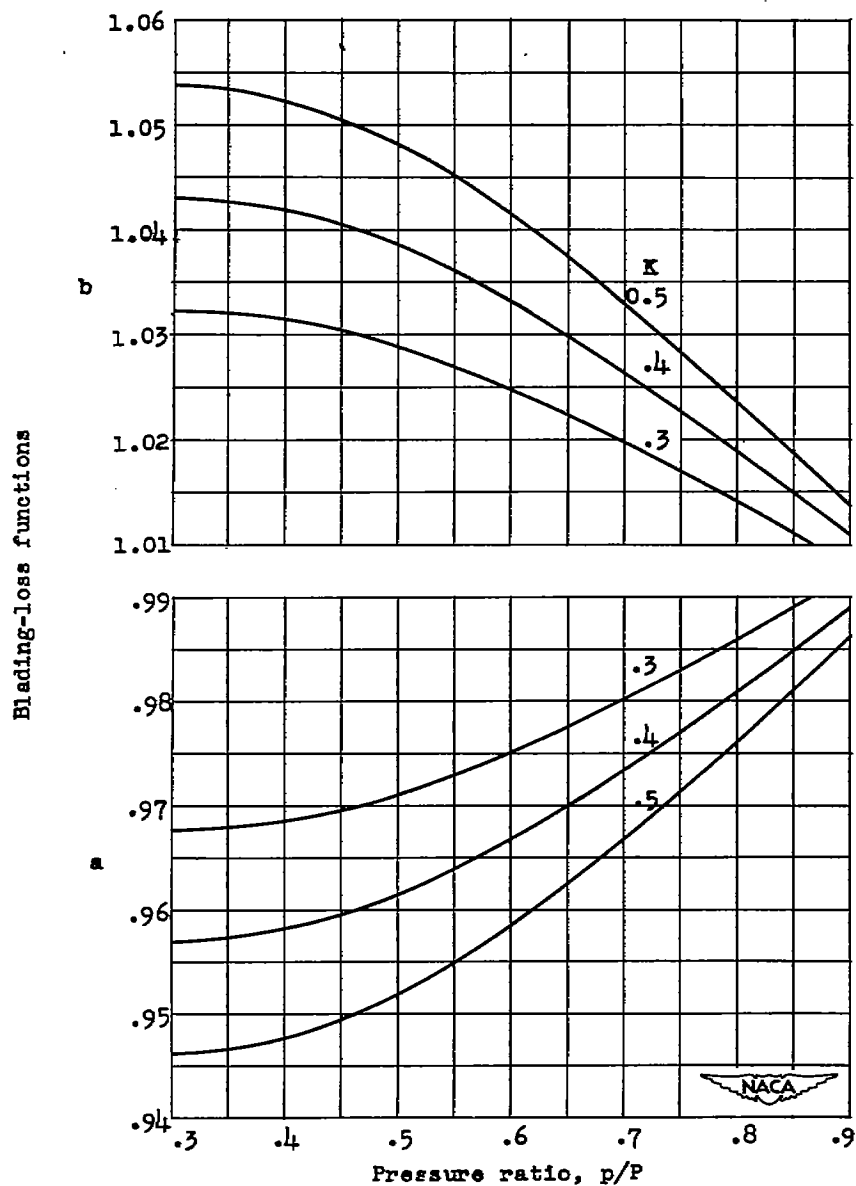
Figure 2. - Variation of blading-loss functions a and b with ratio of static to total pressure for several values of blading-loss parameter K. $\gamma = 1.34$.

$$a = 1 - K \left(\frac{p}{P}\right)^{\frac{1}{\gamma}} \left[1 - \left(\frac{p}{P}\right)^{\frac{\gamma-1}{\gamma}} \right] ;$$

$$b = 1 + K \left(\frac{p}{P}\right)^{\frac{1}{\gamma}} \left[1 - \left(\frac{p}{P}\right)^{\frac{\gamma-1}{\gamma}} \right] .$$
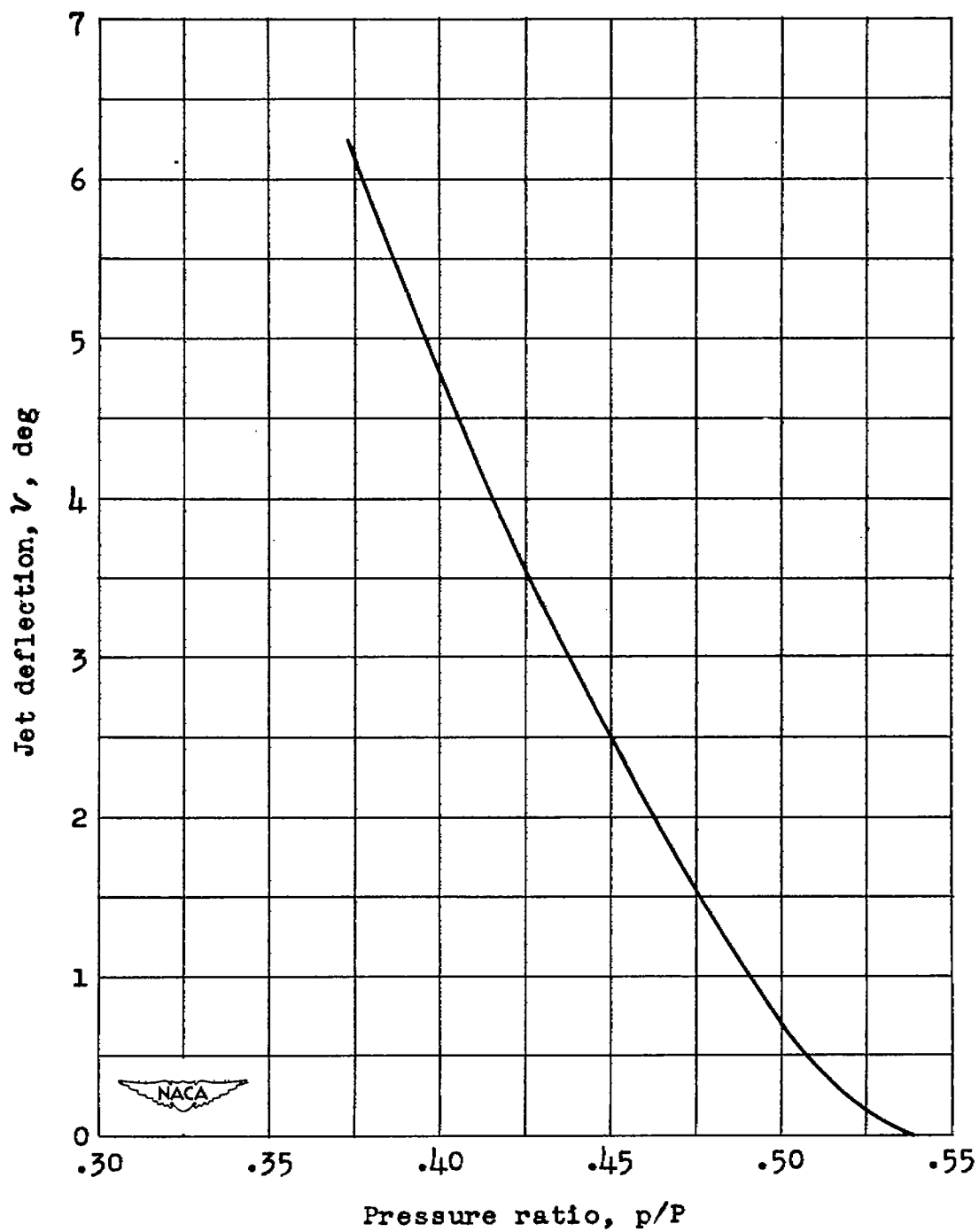
Figure 3. - Variation of jet deflection with ratio of static-to-total pressure in region beyond blade exit. γ = 1.34.

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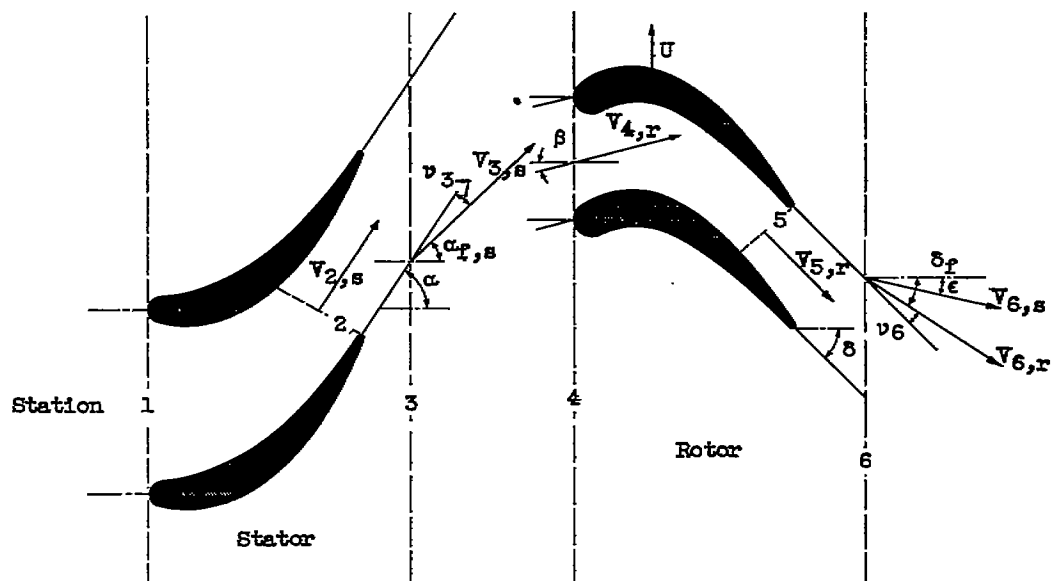
Figure 4. - Schematic diagram of turbine.
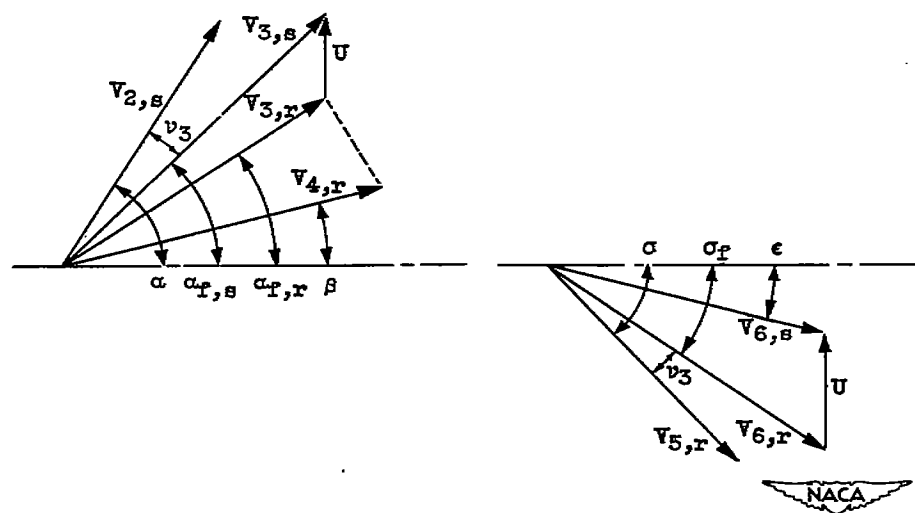
Figure 5. - Turbine-velocity diagram.
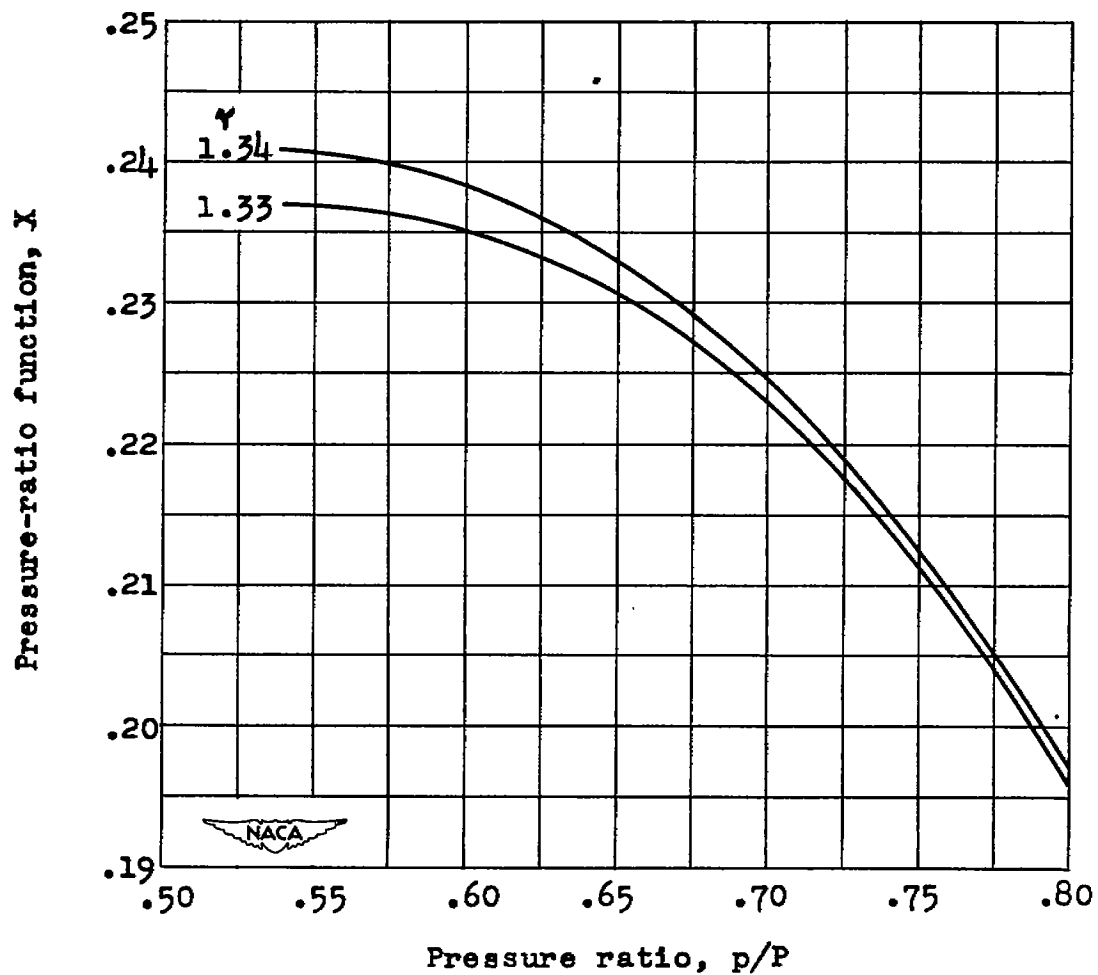
Figure 6. - Variation of pressure-ratio function X
with ratio of static-to-total pressure.

$$X = \left[\left(\frac{p}{P}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{P}\right)^{\frac{\gamma+1}{\gamma}} \right]^{\frac{1}{2}}$$

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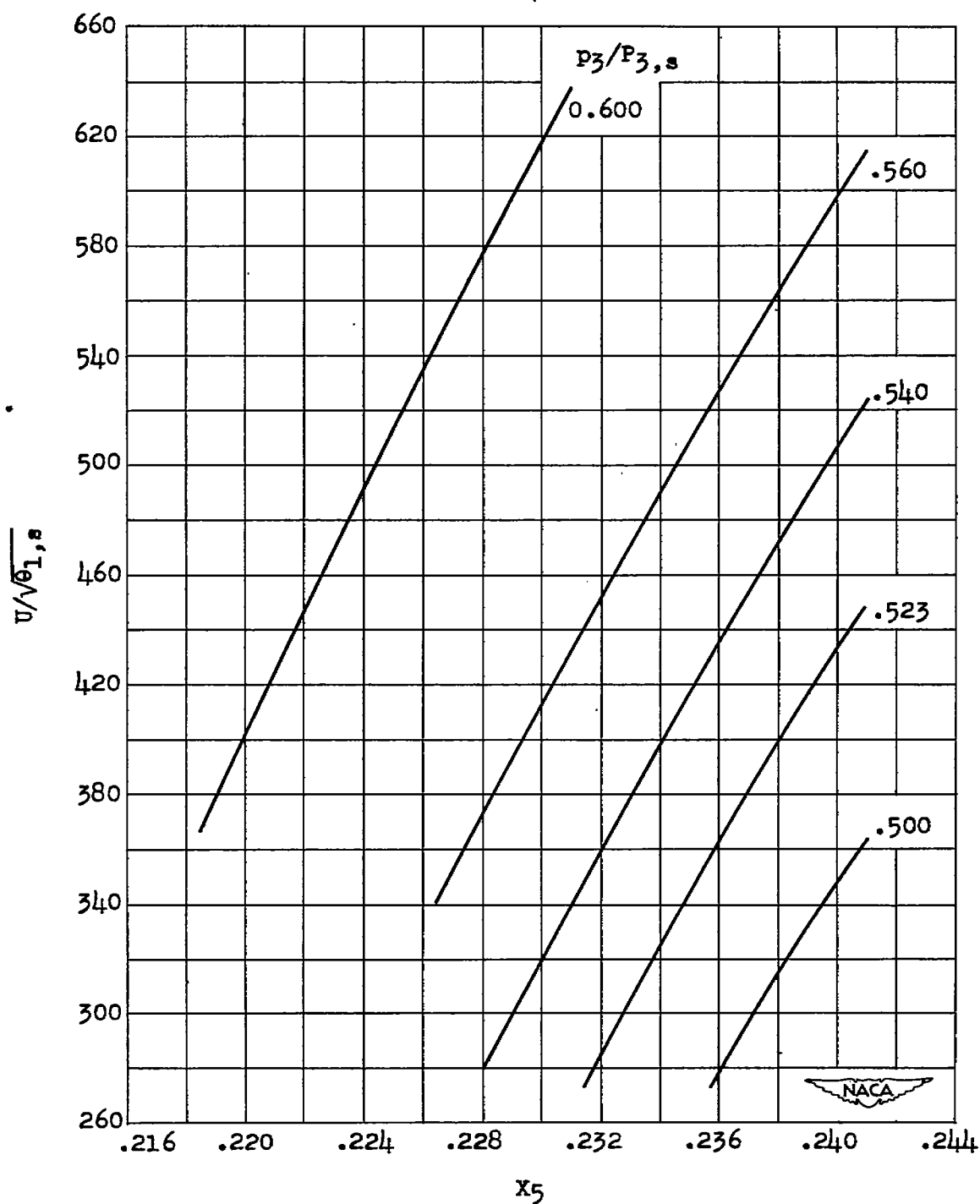
Figure 7. - Determination of rotor-choking values of $U/\sqrt{\theta_{1,s}}$ for several values of $p_3/P_{3,s}$. Turbine of typical example; K = 0.3; C = 0.98.
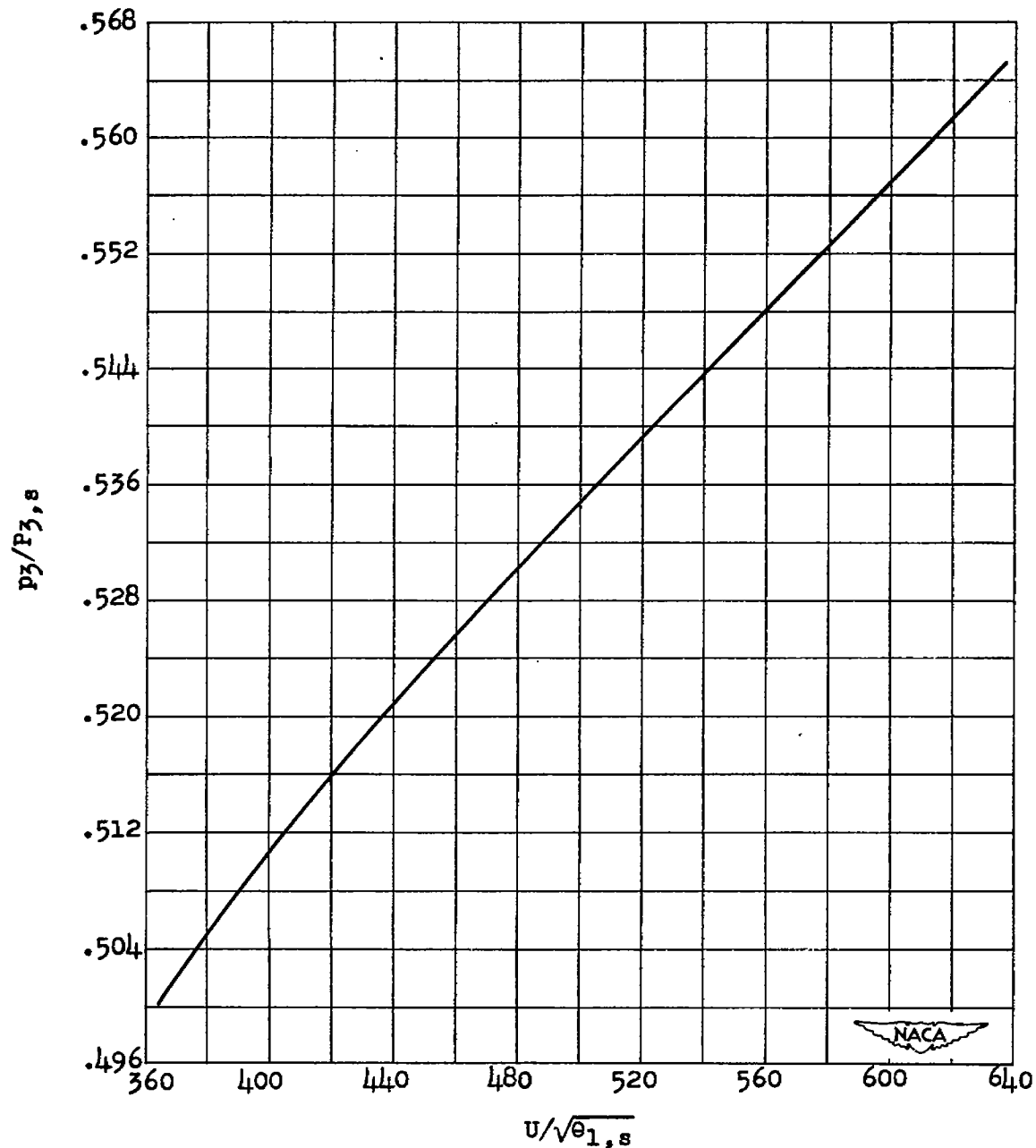
Figure 8. - Values of $p_3/P_{3,s}$ and $U/\sqrt{\theta_{1,s}}$ that correspond
to rotor choking. Turbine of typical example; K = 0.3;
C = 0.98.
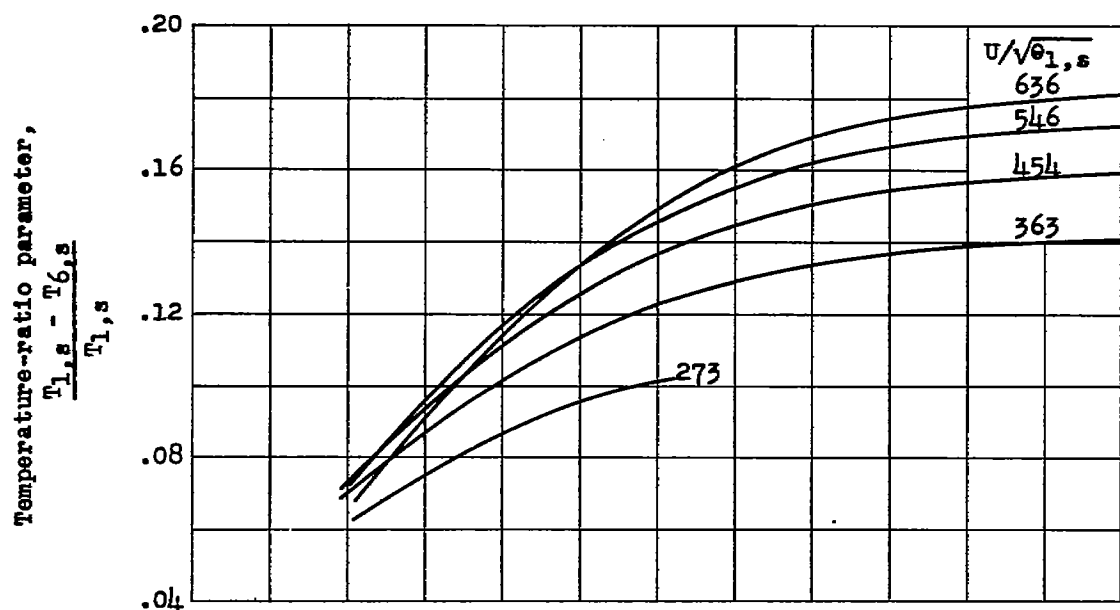
(a) Temperature-ratio parameter.
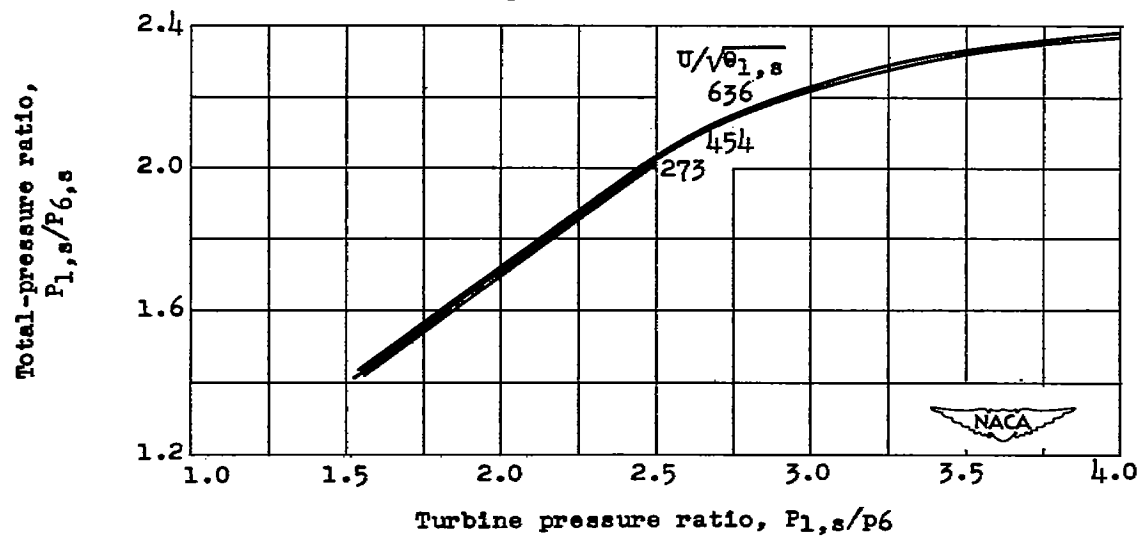
(b) Total-pressure ratio.

Figure 9. - Performance parameters for turbine of typical example.
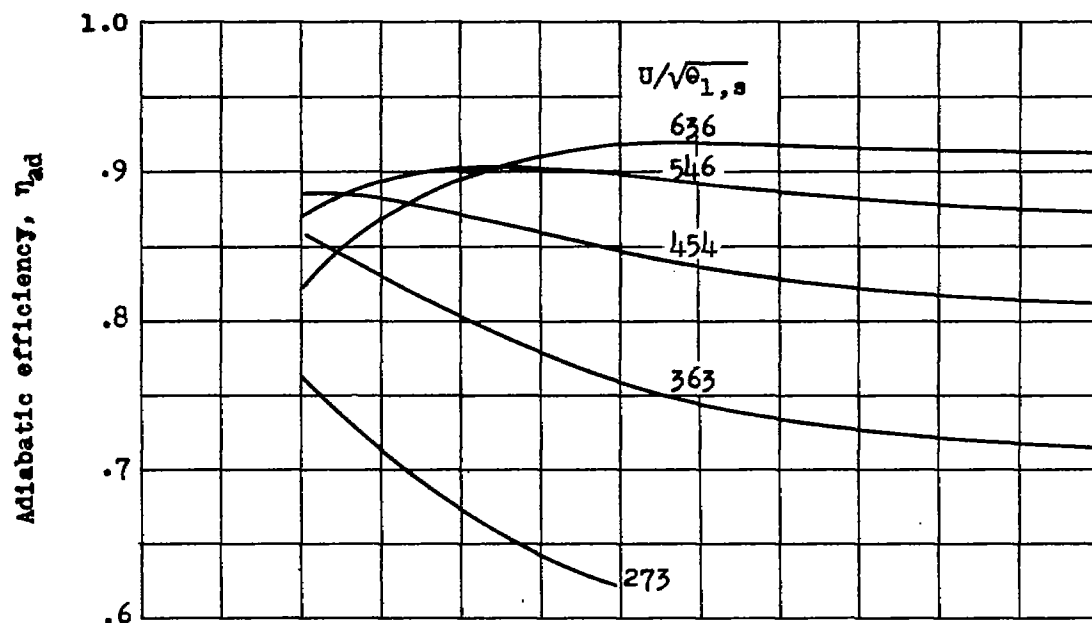
(c) Adiabatic efficiency.
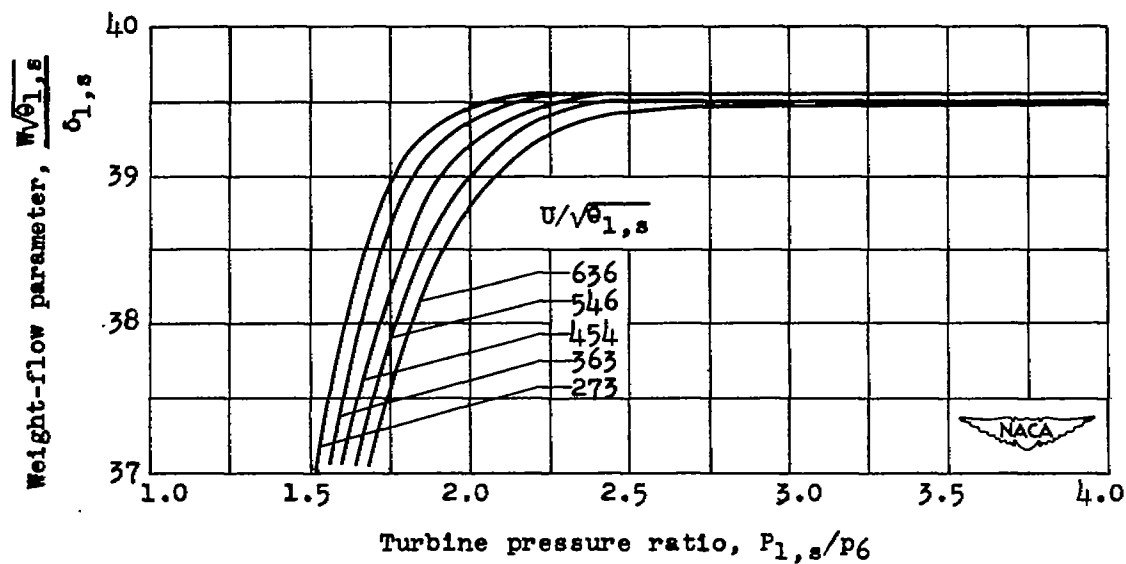
(d) Weight-flow parameter.

Figure 9. - Continued.   Performance parameters for turbine of
                         typical example.
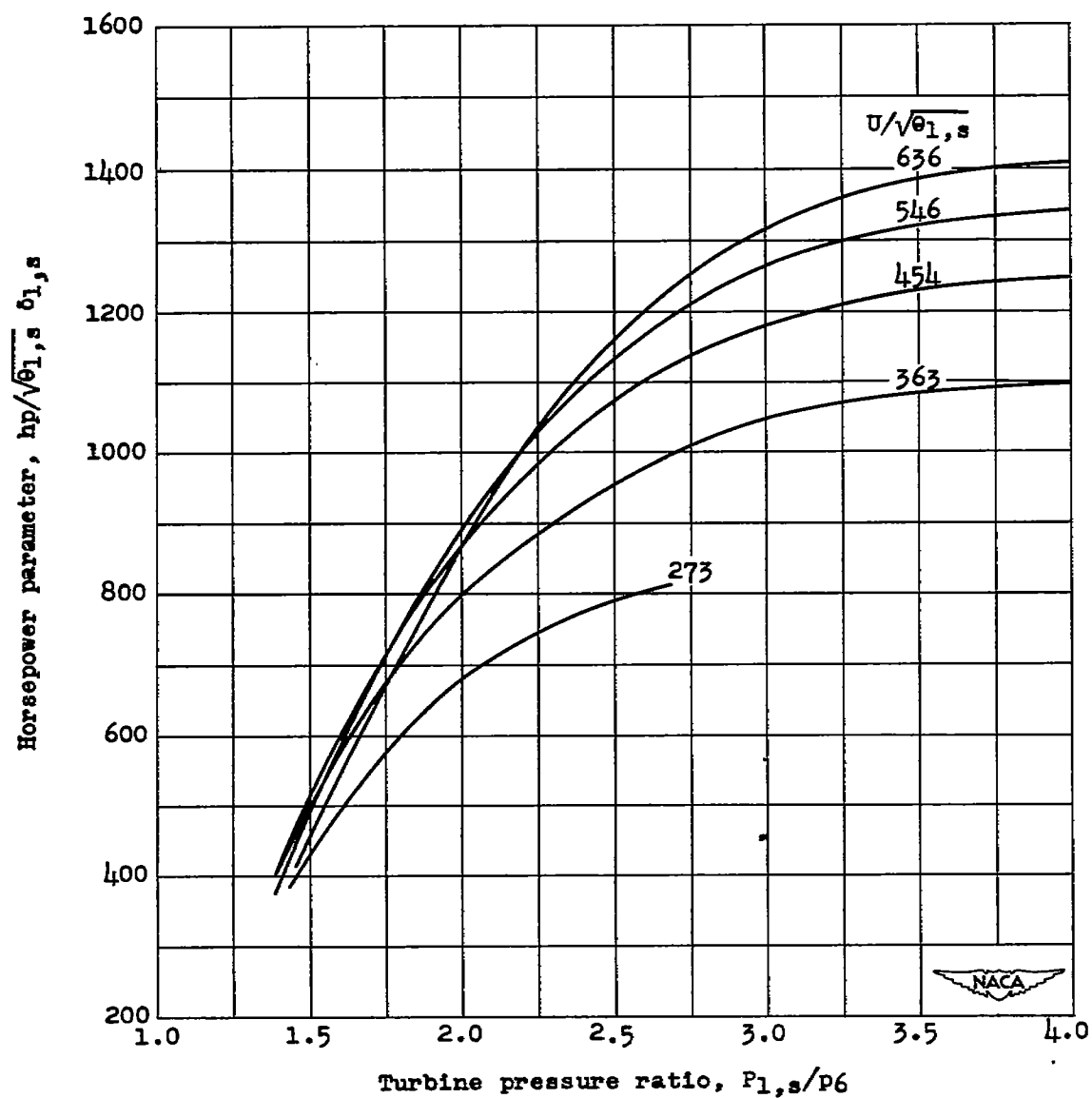
(e) Horsepower parameter.

Figure 9. - Concluded. Performance parameters for turbine of
typical example.
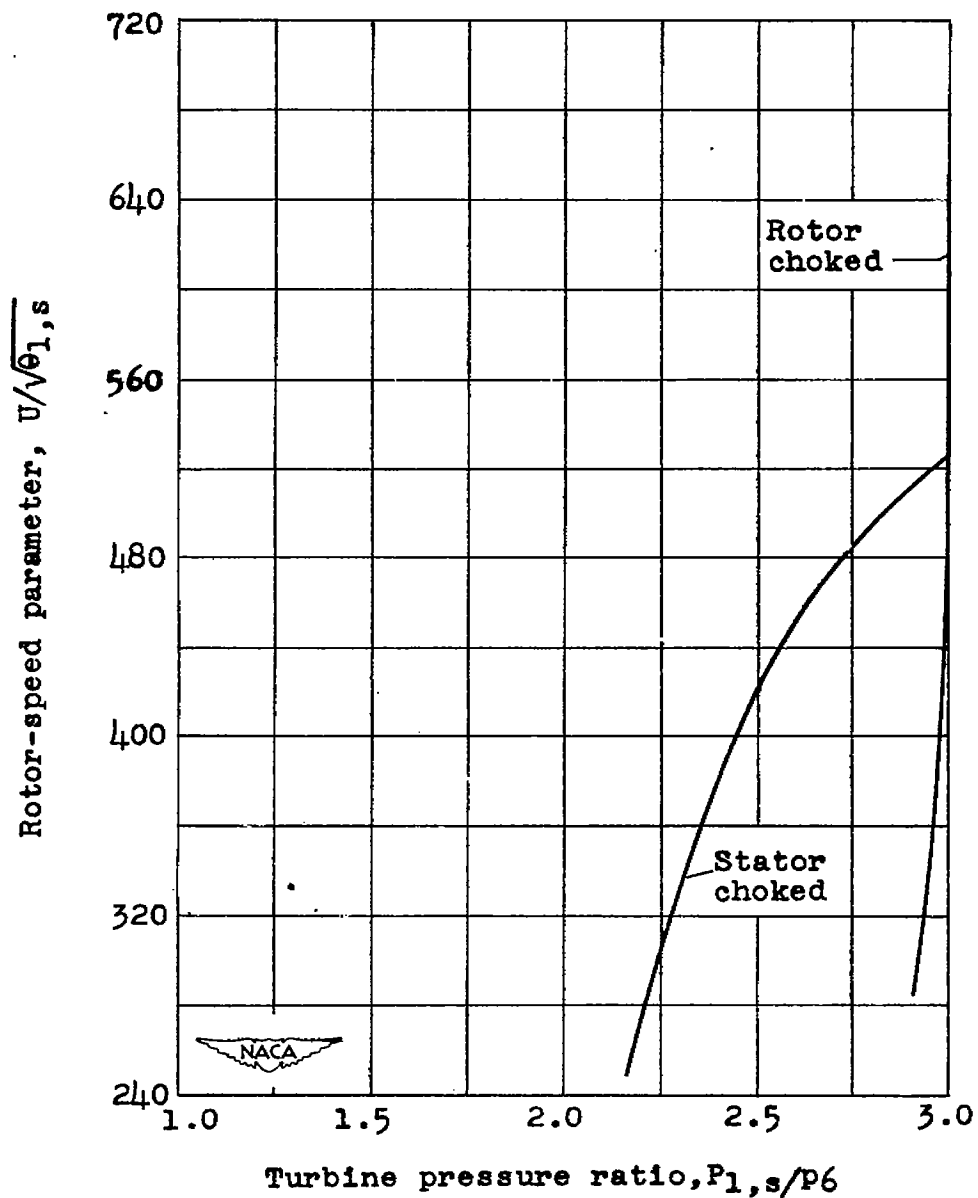
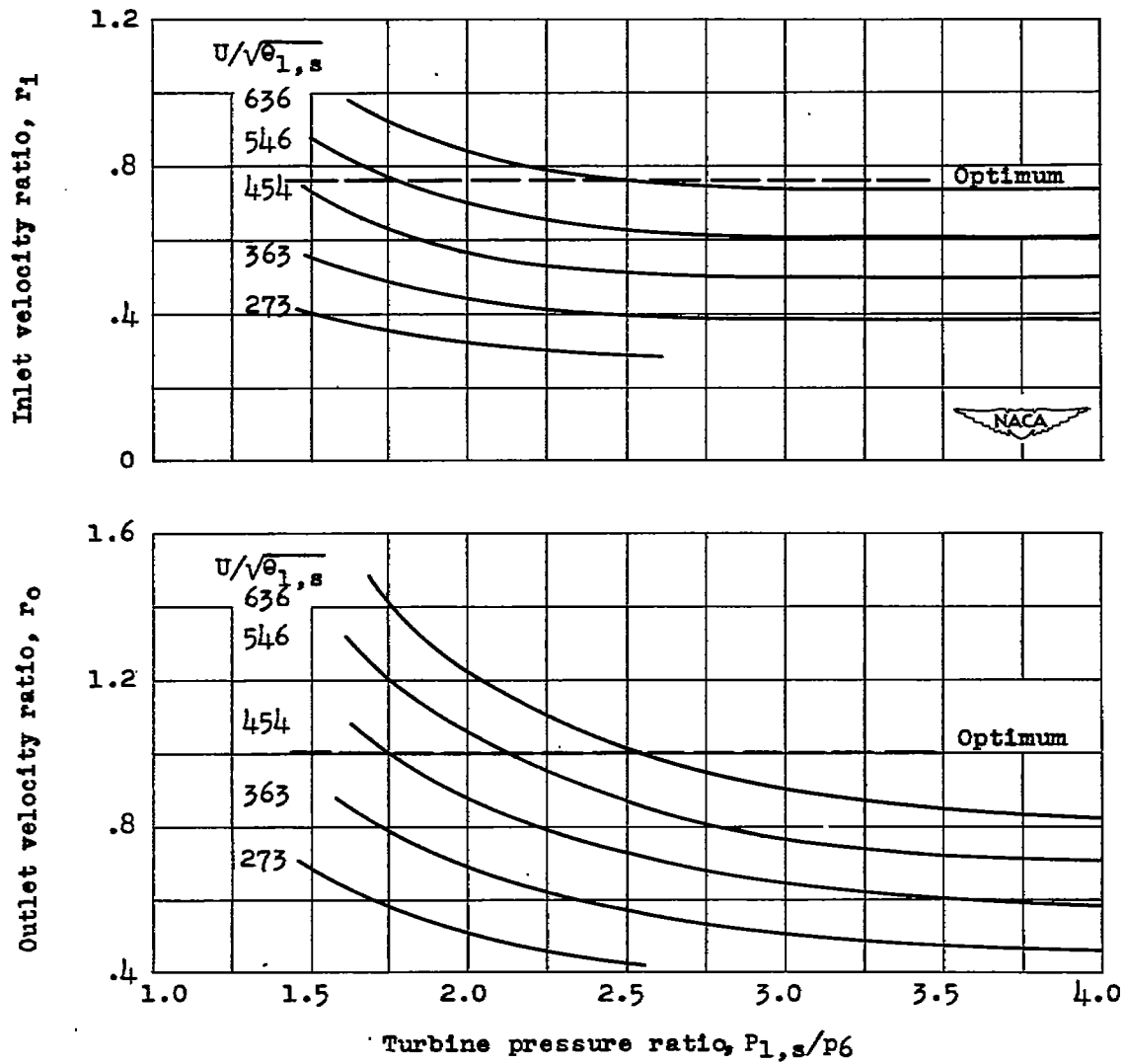Figure 10. - Conditions for turbine choking in turbine of typical example. K = 0.3; C = 0.98

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Figure 11. - Variation of inlet and outlet velocity ratios for
turbine of typical example.   K = 0.3; C = 0.98.

(a) Temperature-ratio parameter.

(b) Adiabatic efficiency.
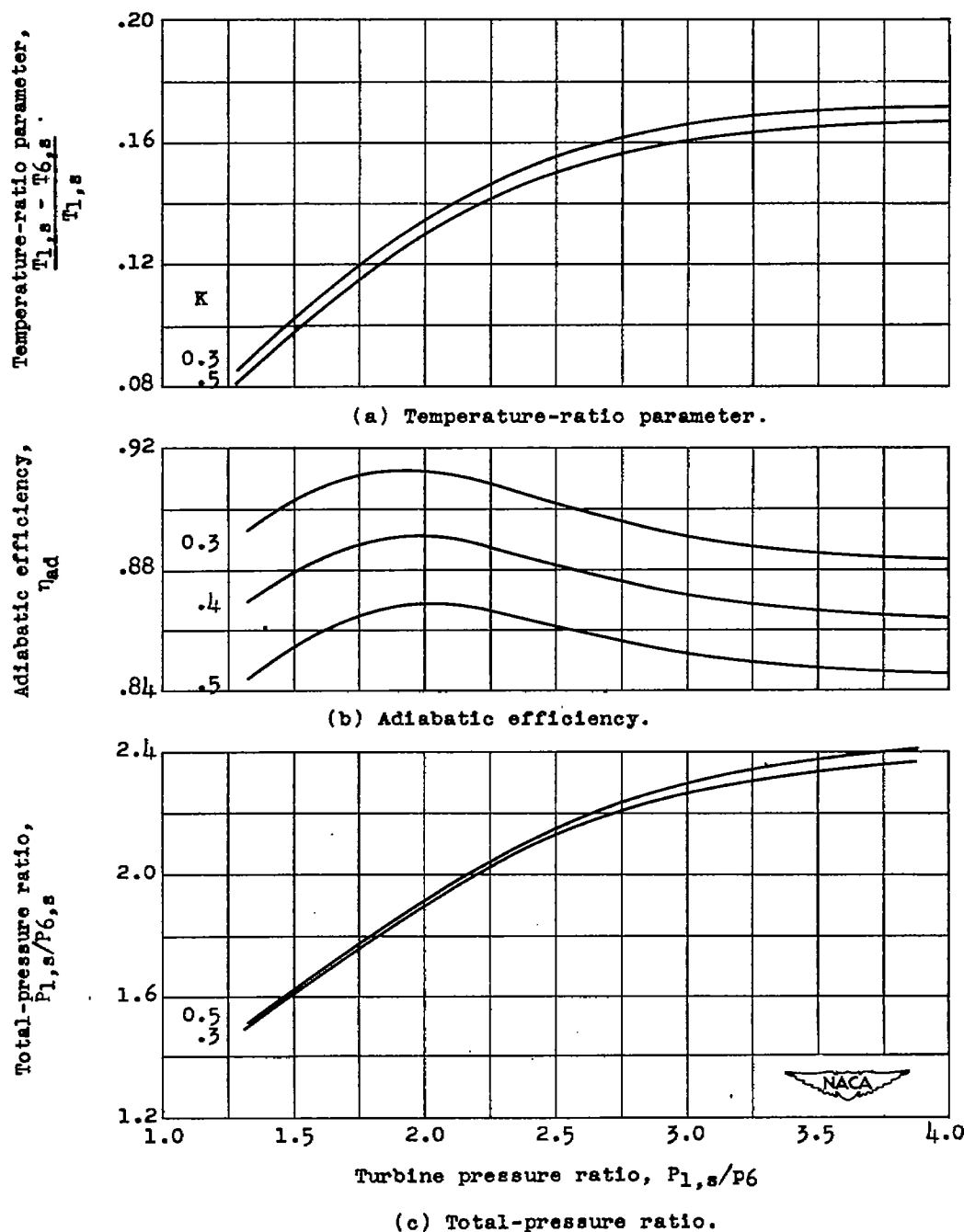
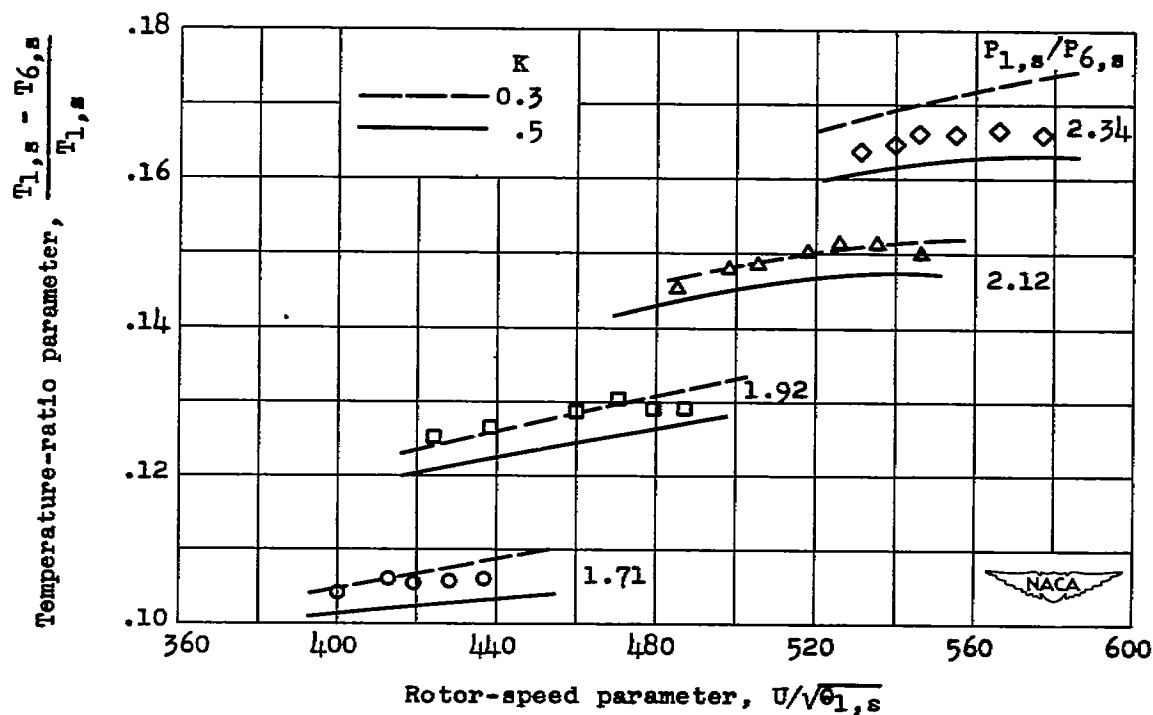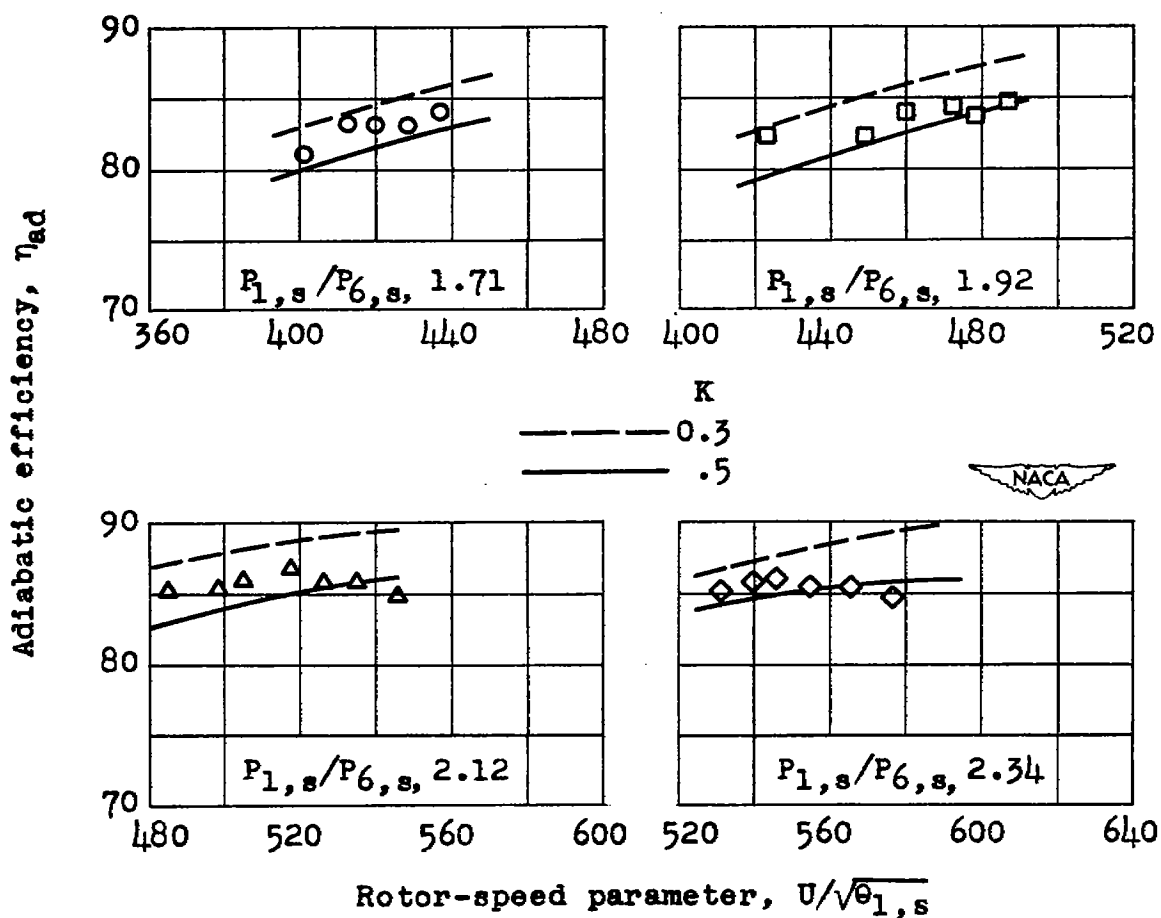Turbine pressure ratio, $P_{1,s}/P6$

(c) Total-pressure ratio.

Figure 12. - Effect of blading-loss parameter K on turbine performance parameters. Turbine of typical example; $U/\sqrt{\theta}_{1,s} = 546$; C = 0.98.

(a) Variation of temperature-ratio parameter with
rotor-speed parameter.

Figure 13. - Comparison between theoretical and experimental
results for turbine of typical example.

(b) Variation of adiabatic efficiency with
rotor-speed parameter.

Figure 13.- Concluded. Comparison between theoretical
and experimental results for turbine of typical
example.